EFFICIENT ESTIMATION ALGORITHM FOR ARMA MODEL FOR COLOURED NOISE

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Abstract

In this paper, a modified estimation algorithm has been developed refers to Covariance Shaping Least Square (CSLS) estimation based on the quantum mechanical concepts and constraints. The algorithm has been applied to Auto Regressive Moving Average (ARMA models with various parameter values. The same models can be applied with Colored Noise which estimates the bias in the parameter and the validity of the uncertainty estimates refers to Monte Carlo simulation. Building upon the problem of optimal quantum measurement design, we develop and discuss the performance of the CSLS estimator with the measure of mean square error (MSE) and it is compared with Least Square, James Stein, Shrunken and Ridge estimators for different applications. It proved that the CSLS estimator has low MSE and performed efficiently better than others at low to moderate, even when noise variation is having robust Signal to Noise Ratio (SNR).

Keywords: ARMA, Monte Carlo Simulation, Covariance Shaping, Least Square Estimation, Signal to Noise ratio.

1. Introduction

The development in the field of signal processing is tremendous and quantum signal processing in particular has motivated the rigorous growth and research in the past few decades[18,14 and 6].The estimation using digital signal processing concepts have been the research area in the recent past. The quantum mechanical concepts have been shown more interest in the signal analysis due to its inherent properties [7, 8 and 17]. The introduction of Quantum mechanical concepts, which rely on estimation almost entirely on some of signal processing algorithms that are implemented with various techniques. In many DSP applications we don't have complete or perfect knowledge of the signals we wish to process. We are faced with many unknowns and uncertainties like noisy measurements and unknown signal Parameters [6].

We address the problem of estimating an unknown parameter vector x in a linear model

$$Y = Hx+w \qquad (1)$$

subject to the a-priori information that the true parameter vector x, $H = [H_1][H_2]….[H_m]$ is a known $n \times m$ matrix and w is a Zero-mean random vector with covariance C_w. It is well known that among all possible unbiased linear estimators, the LS estimator minimizes the variance [7]. However, this does not imply that the resulting variance or mean-squared error (MSE) is small, where the MSE of an estimator is the sum of the variance and the squared norm of the bias.

To improve the performance of the LS estimator at low to moderate SNR, we propose a modification of the LS estimate, in which we choose the estimator of x to minimize the total error variance in the observations y, subject to a constraint on the covariance of the error in the estimate of x. The resulting estimator of x is derived as the CSLS estimator.

2. PROBLEM FORMULATION

Various modifications of the LS estimator for the case in which the data model is assumed to hold perfectly have been proposed [8]. Stein [17] showed that MSE is small in the LS estimator for certain parameter values when compared to other estimators. An explicit (nonlinear) estimator with this property, which is referred to as the James–Stein estimator, was later proposed and analyzed [16]. This work appears to have been the starting point for the study of alternatives to LS estimators. Among the more prominent alternatives are the Ridge estimator [19], the shrunken estimator [9] and Stochastic Gaussian Maximum Likelihood (ML) estimator [5] that deals with sub-Gaussian signals. In the estimation of unknown parameter the parameterized structure of the *Maximum A-posteriori Probability (MAP)* estimator with prior Gaussian distribution was designed as an improvement of the *mean squared error (MSE)* over the *least-squares (LS)* estimator [6]. Because of some uncertainties in the prominent alternatives minimum mean-squared error and Maximum A Posteriori (MAP) estimators [26] cannot be used in many cases. The minimum mean-squared error linear estimator does not require this prior density.

The problem of estimating the deterministic parameter vector x in a linear regression model, with the mean squared error (MSE) as the performance measure that can be applicable for both admissible and dominating linear estimators [13]. In the past 30 years attempts have been made to develop linear methods that may be biased but close to the true parameter in MSE sense. These include the Tikhonov regularizer[26],the shrunken estimator[20] and the covariance shaping least square estimator[10].Another recent approach is constrained to a subset and then seek linear minimax estimators that minimize a worst case measure of MSE [22,23,11,12,1,2 and 4].Next the problem of estimating random unknown signal parameters in a noisy linear model is processed [27]. In [28] the problem of estimating an unknown deterministic parameter vector in a linear model with a Gaussian model matrix has been analysed.The maximum likelihood estimator associated with this problem and show that it can be found using a simple line search over a unimodel function which can be efficiently evaluated. We then analyze its performances using Cramer Rao bound.

Specifically, it is achieving the Cramer Rao lower bound for biased estimators [21, 15] when the noise is Gaussian. The efficient estimation algorithm of ARMA model with White Gaussian Noise can be performed in [24] based on the quantum mechanical concepts and constraints. The efficient estimation algorithm of Exponential and other Trigonometric model with White Gaussian Noise can be performed in [25].

Here the problem of estimating an unknown deterministic parameter vector in a linear model with a Coloured noise has been analyzed. The performance of the CSLS estimator with the measure of mean square error (MSE) and it is compared with Least Square, James Stein, Shrunken and Ridge estimators for different applications. The CSLS estimator has a property analogous to the property of the LS estimator. Instead of the traditional mean squared error (MSE) approach, we propose a linear estimator that minimizes the MSE which is averaged over the coloured noise. In which case, the LS estimate may result in a poor estimate. This effect is especially predominant at low to moderate signal-to-noise ratio (SNR) and even at the robust noise variation of SNR values.

3. LEAST SQUARE ESTIMATION

This approach uses short-time FFT to transform the input signal into the spectral domain. Similar to the generation of a spectrogram, the FFT is applied on data collected from a short time frame, which advances in time with some overlap. Threshold detection is performed in each spectral profile to determine whether a signal is present with certain likelihood. When such detection occurs in a number of consecutive frames, the frequencies of the detected peaks are used to determine the starting frequency and chirp rate of the signal by means of linear regression, i.e., the least squares method. The processing sequence of this method is shown in Fig.1.

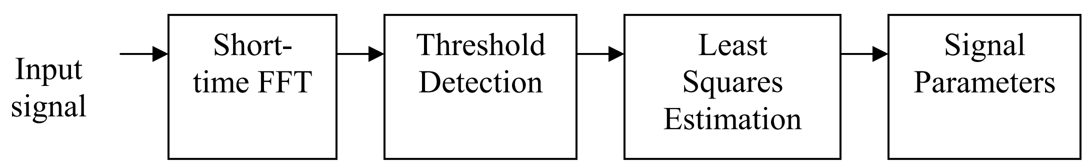

Fig.1 LS method for detection and estimation of signal

The LS estimator resulting variance or trace mean-squared error (MSE) is small, where the MSE of an estimator is the sum of the variance and the squared norm of the bias. In which case, the LS estimate may result in a poor estimate. This effect is especially predominant at low to moderate signal-to-noise ratio (SNR). In linear algebra, the trace of an *n*-by-*n* square matrix *A* is defined to be the sum of the elements on the main diagonal (the diagonal from the upper left to the lower right) of *A*, i.e.,

$$tr(A) = a_{11} + a_{22} + \ldots + a_{nn} = \sum_i a_{ii} \tag{2}$$

where a_{ij} represents the entry on the *i*th row and *j*th column of *A*. Equivalently, the trace of a matrix is the sum of its eigen values, making it an invariant with respect to chosen basis. For an *m*-by-*n* matrix *A* with complex (or real) entries and * being the conjugate transpose, we have

$$tr(A^* A) \geq 0 \tag{3}$$

with equality only if $A = 0$. The assignment

$$\langle A, B \rangle = tr(A^* B) \tag{4}$$

yields an inner product on the space of all complex (or real) *m*-by-*n* matrices. If $m=n$ then the norm induced by the above inner product is called the Frobenius norm of a square matrix. Indeed it is simply the Euclidean norm if the matrix is considered as a vector of length n^2.

4. JAMES-STEIN ESTIMATOR

Suppose θ is an unknown parameter vector of length *m* and let Y be observations of the parameter vector such that $Y \sim N(\theta,\sigma^2 I)$. This is a situation in which a set of parameters is measured and the measurements are corrupted by independent Gaussian noise. Since the noise has zero mean it is very reasonable to use the measurements themselves as an estimate of the parameters and this is equal to least squares estimator. Stein demonstrated that, in terms of mean squared error is suboptimal. The James-Stein estimator is given

$$\hat{\theta}_{js} = \left(1 - \frac{(M-2)\sigma^2}{\|Y\|^2}\right) Y. \tag{5}$$

James and Stein showed that the above estimator dominates $\hat{\theta}_{LS}$ for any, m ≥ 3 meaning that the James-Stein estimator always achieves lower MSE than the least squares estimator. Stein has shown that, for m ≤ 2, the least squares estimator is admissible, meaning that no estimator dominates it.

When three or more unrelated parameters are measured, their total MSE can be reduced by using a combined estimator such as the James-Stein estimator whereas when each parameter is estimated separately, the least squares (LS) estimator is admissible. The response is that the James-Stein estimator always improves upon the total MSE, i.e., the sum of the expected errors of each component.

5. RIDGE ESTIMATOR

The ridge estimator has become the most common method to overcome the weakness of LS estimator. The ridge regression can be obtained by augmenting the the original equation Y=X β +ε and then using the least squares method.

$$\beta(k) = (X^{'} kI)^{-1} X^{'} Y, k > 0 \tag{6}$$

Therefore, augmenting to the original equation introduces more bias to the ridge regression. For this reason, it is desirable to select a small k and thus $X^{'}X$ + kI may still be ill conditioned. It is easy to see that the condition number of $X^{'}X$ + kI is a decreasing function of k.

6. SHRUNKEN ESTIMATOR

The matrix MSE of a shrunken is not bigger than matrix MSE of the LS estimator and satisfied the following matrix inequality.

$$XX'(\beta-1)^2 + \beta^2 (C'\Sigma_v^{-1} C)^{-1} \le (C'\Sigma_v^{-1} C)^{-1} \tag{7}$$

This requires that β be chosen such that

$$\frac{X'(C'\Sigma_v^{-1}C)X - 1}{X'(C'\Sigma_v^{-1}C)X + 1} \le \beta \le 1 \tag{8}$$

Notice that for β=1, the shrunken estimator coincides with the LS estimator and for

$$\beta = \frac{X'(C'\Sigma_v^{-1}C)X - 1}{X'(C'\Sigma_v^{-1}C)X + 1} \tag{9}$$

gives a bias estimate with the same MSE of LS.

7. COVARIANCE SHAPING LEAST-SQUARES ESTIMATION

The CSLS estimator is directed at improving the performance of the LS estimator at low to moderate SNR by choosing the estimate of x to minimize the total error variance in y subject to a constraint on the covariance of the error in the estimate so that we control the dynamic range and spectral shape of the covariance of the estimation error.

The CSLS estimate of x, which is denoted $\hat{x}_{CSLS}$, is chosen to minimize the total variance of the weighted error between $\hat{y} = H\hat{x}_{CSLS} = HGy$ and y, subject to the constraint that the covariance of the error in the estimate $\hat{x}_{CSLS}$ is proportional to a given covariance matrix R. The covariance of y is equal to C_w so that the covariance of $\hat{x}_{CSLS}$, which is equal to the covariance of the error in the estimate $\hat{x}_{CSLS}$ is given by GC_wG^*. Thus $\hat{x}_{CSLS} = Gy$, is chosen to minimize

$$\xi_{CSLS} = \sum \left((y' - HGy')^* Cw^{-1} (y' - HGy') \right) \tag{10}$$

subject to

$$GC_w G^* = c^2 R \tag{11}$$

where $y' = y - E(y)$, R is a given covariance matrix and $c > 0$ is a constant that is either specified in advance or chosen to minimize the error.

8. IMPLEMENTATION OF CSLS ESTIMATION

8.1 ARMA MODEL

Suppose we are given a finite segment of noisy measurements of an ARMA signal $x[l]$, which is defined by equation (12)
The z -transform of $x[l]$ is

$$x[l] = \sum_{k=1}^{p} a_k x[l-k] + \sum_{k=0}^{q} b_k \delta[l-k] \tag{12}$$

$$X(z) = \frac{b_0 + b_1 z^{-1} + \dots\dots + b_q z^{-q}}{1 + a_1 z^{-1} + \dots\dots\dots a_p z^{-p}} = B(z)H(z) \tag{13}$$

for some coefficients a_k and b_k, where $q \le p$. The coefficients a_k are the AR parameters of $x[l]$, and the coefficients b_k are the MA parameters. The $B(z)$ denotes the numerator polynomial and $H(z)$ denotes the inverse of the denominator polynomial. The problem then is to estimate the AR and MA parameters from the data $y[0], \dots\dots\dots y[n-1]$ where $y[l] = x[l] + w[l]$

$$= \sum_{k=1}^{p} a_k x[l-k] + \sum_{k=0}^{q} b_k \delta[l-k] + w[l] \qquad 0 \le l \le n-1 \tag{14}$$

Here, $w[l]$ represents a combination of measurement noise and modeling error. In the simulations below, $w[l]$ is chosen as a zero-mean Gaussian noise process with variance σ^2.

Various methods exist for estimating the ARMA parameters based on different applications of LS estimation. A popular method is to estimate the AR parameters using the modified Yule–Walker equations and then use these estimates in combination with Shanks' method to estimate the MA parameters. We use this method as a basis for comparison with our method. It follows that

$$y[l] = \sum_{k=1}^{p} a_k x[l-k] + w[l] \qquad q < l \le n-1 \tag{15}$$

We now use the method proposed in [27] to estimate the AR parameters a_k. Since we do not have access to the clean data $x[l-k]$, we estimate a_k by substituting $y[l-k]$ instead of $x[l-k]$ in the following equation. Then, a denoting the vector with components a_k, $1 \le k \le p$, y denoting the data vector with components $y[l]$, $p \le l \le n-1$, w denoting the vector with components $w[l]$, $p \le l \le n-1$ and

$$H_{AR} = \begin{bmatrix} y(p-1) & y(p-2) & \dots & y(0) \\ y(p) & yp(-1) & \dots. & y(1) \\ : & : & \dots. & : \\ y(n-2) & y(n-3) & \dots. & y(n-p-1) \end{bmatrix} \tag{16}$$

We have that $y \approx H_{AR} a + w$.

The LS estimate of the AR parameters is

$$\hat{a}_{LS} = \left(H_{AR} * H_{AR}\right)^{-1} H_{AR} * y \quad (17)$$

The CSLS estimate of the AR parameters is

$$\hat{a}_{CSLS} = \hat{c}\left(RH_{AR} * H_{AR}\right)^{-1/2} RH_{AR} * y \quad (18)$$

We now use these estimates of a to estimate the MA parameters using Shanks' method. Specifically, let $e[l] = y[l] - h[l] * b[l]$, where $h[l]$ is the impulse response of the filter with z-transform $H(z)$ which is computed using the estimates of the AR parameters and $b[l]$ is the (unknown) impulse response of the filter with z-transform $B(z)$. Shanks proposed estimating the unknown sequence $b[l]$ by minimizing $\sum_{l=0}^{n} e^2[l]$. With e denoting the error vector with components $e[l]$, we have that $e = y - H_{MA} b$ where b is the vector with components $b_k, 1 \le k \le q$. y is the data vector with components $y[l], 0 \le l \le n-1$ and

$$H_{MA} = \begin{bmatrix} h(0) & 0 & .... & 0 \\ h(1) & h(0) & .... & 0 \\ : & : & .... & : \\ h(n-1) & h(n-2) & .... & h(n-q) \end{bmatrix} \quad (19)$$

so that Shank's method reduces to a LS problem. The LS estimator of the MA parameters is then

$$\hat{b}_{LS} = \left(H_{MA} * H_{MA}\right)^{-1} H_{MA} * y \quad (20)$$

where H_{MA} is computed using the LS estimate. We can modify Shanks' estimator by using the CSLS estimator of the parameters b, which leads to the estimator

$$\hat{b}_{CSLS} = \hat{c}\left(RH_{MA} * H_{MA}\right)^{-1/2} RH_{MA} * y \quad (21)$$

where $\hat{c}$ is given by (9), and now, H_{MA} is computed using the CSLS estimate $\hat{a}_{CSLS}$ of (11).
The estimates of the uncertainty matrix associated with the fitted parameters are based on a linearization about the solution and could give misleading results. The aim of the Monte Carlo simulation is to estimate the bias in the parameter estimates and the validity of the uncertainty estimates.

9. RESULTS AND DISCUSSION

Complex model parameters like sinusoidal, cosine, exponential, ARMA can be estimated using CSLS Estimation for colored noise. The data is, $y[l] = A(\cos\theta)$, $y[l] = a_1 e^{s_1 l} + a_2 e^{s_2 l}$ where θ is varies from 0 to 2π. $y[l] = A(\sin c(\theta))$ is also considered here. The unknown amplitude is measured through various estimation methods the signals will be tested with all estimators. Finally performances of MSE are observed. The amplitude A was measured through the various estimation methodology. Finally the error performances of all estimators are observed and the plots and tables are given below.

9.1 ARMA (1, 0) Model Estimation for Coloured Noise

For this model, number of AR parameter is 1, and the number of MA parameter is 0. Assign the values with a_1 = 0.9, b_0 =1. To evaluate the performance of estimators the MSE values are plotted in estimating the AR parameters using $\hat{a}_{CSLS}$ for averaged over 2000 noise realizations, as a function of -10logσ^2, where σ^2 is the noise variance. The MSE of the CSLS estimator decreases with for low SNR and then converges to a constant in the high SNR limit. The CSLS estimator is also compared with the shrunken, ridge and James Stein estimator described in this section.

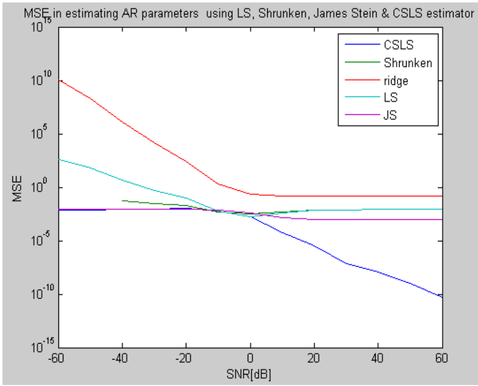

Figure 1(a) MSE in estimating AR parameter in ARMA (1,0) model for coloured noise

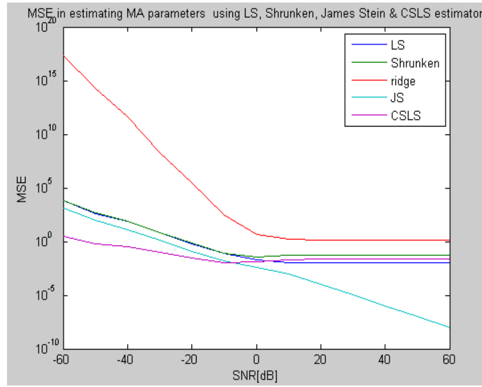

Figure 1 (b) MSE in estimating MA parameter in ARMA(1,0) model for coloured noise

Table 1 MSE in estimating AR parameter in ARMA (1, 0) model for coloured noise (-60 db to +60 db)

SNR(db)	MSE-LS	MSE-JS	MSE-CSLS	MSE-SE	MSE-RE
-60	384.5631	0.008078	0.007452	-0.33864	1.16E+10
-50	67.46822	0.00806	0.007593	-0.07769	1.88E+08
-40	4.186321	0.008142	0.009039	0.053395	1209907
-30	0.498164	0.00822	0.009538	0.030112	13534.31
-20	0.105707	0.008429	0.010344	0.019293	274.3308
-10	0.005543	0.007373	0.006352	0.004756	1.963452
0	0.001684	0.003756	0.001684	0.002885	0.219484
10	0.004062	0.001332	5.63E-05	0.005039	0.158497
-20	0.006583	0.000973	3.38E-06	0.006985	0.135957
-30	0.007596	0.000921	6.99E-08	0.007735	0.141641
40	0.007939	0.000918	1.14E-08	0.007984	0.140412
50	0.008049	0.000918	1.06E-09	0.008063	0.140442
60	0.008084	0.000918	5.33E-11	0.008088	0.140637

Therefore, MSE values are much lower for CSLS estimator compared with other estimators such as LS, Ridge, Shrunken estimators for SNR values ranging from -60 db to +60db. In the estimation AR parameter in the model ARMA (1, 0), at SNR -60 dB level, the CSLS estimator has high MSE compare to LS, Shrunken estimator. But at this range Ridge estimator give maximum error.

The MSE values are plotted in estimating the MA parameters using $\hat{b}_{CSLS}$ averaged over 2000 noise realizations, as a function-10log σ^2, where σ^2 is the noise variance. In this case, the observation is that the CSLS estimator performs better than the least square estimator and shrunken estimator for all SNR. For SNR values up to roughly 25 db to 30 db, the CSLS estimator also performs better than the ridge estimator and James Stein Estimator.

Table 2 MSE in estimating MA parameter in ARMA (1, 0) model for coloured noise (-60 db to +60 db)

SNR(db)	MSE-CSLS	MSE-JS	MSE-LS	MSE-SE	MSE-RE
-60	1396.778	3.003636	6251.155	6608.507	2E+17
-50	87.26821	0.618428	397.5424	453.0712	1.79E+14
-40	12.67568	0.314437	74.29257	79.90321	4.04E+11
-30	1.261678	0.091097	6.210334	6.628014	1.99E+08
-20	0.130259	0.030604	0.667203	0.740205	310005.2
-10	0.014696	0.010815	0.074489	0.081266	309.7922
0	0.003823	0.012158	0.017769	0.039104	4.657099
10	0.00088	0.021231	0.011316	0.051383	1.81257
-20	9.9E-05	0.022496	0.010425	0.051156	1.372911
-30	1E-05	0.022819	0.009937	0.051943	1.395399
40	1E-06	0.022802	0.009992	0.051816	1.381771
50	1E-07	0.022805	0.010004	0.051827	1.382376
60	1E-08	0.022811	0.01	0.051851	1.384246

MSE are gradually decrease upto 0 db level and increase lightly and reach the LS value at 20 db level. Here CSLS estimator gets better result up to the SNR levels -10 db. After the 0 level of SNR, CSLS estimator yields minimum variance with others. The MSE values are also calculated for different types of estimators such as CSLS, LS, Ridge, Shrunken and James Stein estimators particularly for 0 db to 10 db where the noise variation is robust in this range.

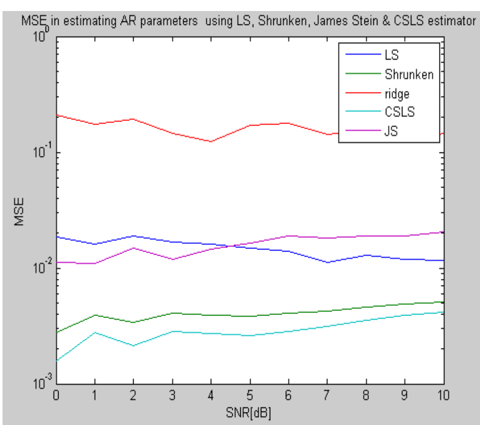

Figure 2 (a) MSE in estimating AR parameter in ARMA (1, 0) model for coloured noise

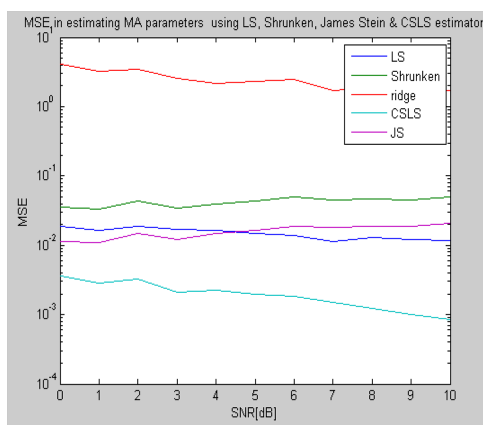

Figure 2 (b) MSE in estimating MA parameter in ARMA (1, 0) model for coloured noise

Table 3 MSE in estimating AR parameter in ARMA (1, 0) model
for coloured noise (0 db to 10 db)

SNR(db)	MSE-CSLS	MSE-LS	MSE-SE	MSE-RE	MSE-JS
0	0.00157	0.018472	0.00278	0.207354	0.011197
1	0.002786	0.016222	0.003873	0.173994	0.010872
2	0.002148	0.018874	0.003371	0.192942	0.01485
3	0.002858	0.016885	0.004053	0.145953	0.011959
4	0.002706	0.016213	0.003906	0.123184	0.01464
5	0.002598	0.014918	0.003822	0.170584	0.016497
6	0.002846	0.014045	0.00403	0.177679	0.018804
7	0.003131	0.01124	0.004248	0.142353	0.018272
8	0.003502	0.012744	0.004582	0.154661	0.019011
9	0.003883	0.011927	0.004877	0.128608	0.018969
10	0.004131	0.011617	0.005087	0.145716	0.020425

The MSE values are plotted in estimating the AR and MA parameters and the simulation results are plotted in Figure 4.13(a) and Figure 4.13(b). The results suggests that the CSLS approach can give a lower MSE for all SNR values upto 10 db compared with these estimators.

Table 4 MSE in estimating MA parameter in ARMA (1, 0) model
for coloured noise (0 db to 10 db)

SNR(db)	MSE-CSLS	MSE-LS	MSE-SE	MSE-RE	MSE-JS
0	0.003663	0.018472	0.035067	4.072859	0.011197
1	0.002849	0.016222	0.033386	3.287389	0.010872
2	0.003226	0.018874	0.043524	3.487484	0.01485
3	0.00213	0.016885	0.034066	2.56032	0.011959
4	0.002218	0.016213	0.038861	2.127858	0.01464
5	0.001961	0.014918	0.044068	2.326647	0.016497
6	0.00182	0.014045	0.049166	2.476554	0.018804
7	0.001489	0.01124	0.044921	1.705227	0.018272
8	0.001217	0.012744	0.04728	1.890619	0.019011
9	0.001005	0.011927	0.045225	1.509278	0.018969
10	0.000856	0.011617	0.049003	1.699111	0.020425

Conclusion

The CSLS estimator has a property analogous to the property of the LS estimator. Specifically, it is shown to achieve the Cramer- Rao lower bound (CRLB) for biased estimators when the noise is Gaussian. This implies that for Gaussian noise, there is no linear or nonlinear estimator with a smaller variance, or MSE, and the same bias as the CSLS estimator. The algorithm developed has been applied to different a model which gives efficient MSE values. For the optimal quantum measurement design, we observed the performance of the CSLS estimator and it is compared with LS, Shrunken and Ridge and James-Stein estimators for different applications and it proved that the CSLS estimator performed significantly better than others at low to moderate SNR.

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