AN OPTIMIZED GLOBAL SYNCHRONIZATION ON SDDCN

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Abstract: The complex networks have been gaining increasing research attention because of their potential applications in many real-world systems from a variety of fields such as biology, social systems, linguistic networks, and technological systems. In this paper, the problem of stochastic synchronization analysis is investigated for a new array of coupled discrete time stochastic complex networks with randomly occurred nonlinearities (RONs) and time delays. The discrete-time complex networks under consideration are subject to: 1) stochastic nonlinearities that occur according to the Bernoulli distributed white noise sequences; 2) stochastic disturbances that enter the coupling term, the delayed coupling term as well as the overall network; and 3) time delays that include both the discrete and distributed ones. Note that the newly introduced RONs and the multiple stochastic disturbances can better reflect the dynamical behaviors of coupled complex networks whose information transmission process is affected by a noisy environment. By constructing a novel Lyapunov-like matrix functional, the idea of delay fractioning is applied to deal with the addressed synchronization analysis problem. By employing a combination of the linear matrix inequality (LMI) techniques, the free-weighting matrix method and stochastic analysis theories, several delay-dependent sufficient conditions are obtained which ensure the asymptotic synchronization in the mean square sense for the discrete-time stochastic complex networks with time delays. The criteria derived are characterized in terms of LMIs whose solution can be solved by utilizing the standard numerical software. While these solvers are significantly faster than classical convex optimization algorithms, it should be kept in mind that the complexity of LMI computations remains higher than that of solving, say, a Riccati equation. For instance, problems with a thousand design variables typically take over an hour on today's workstations. However, this thesis proposes LMI optimization technique to solve this problem. The advantage of the proposed approach is that resulting stability criterion can be used efficiently via existing numerical convex optimization algorithms such as the interior-point algorithms for solving LMIs

The experimental results show that synchronization using optimized LMI always performs better than LMI.This template is designed for all publications of TMRF Kolhapur. The abstract should summarize the context, content and conclusions of the paper in less than 150 words. It should not contain any reference citations or displayed equations.

Keywords: Complex Networks; Linear Matrix Inequalities; Synchronization; Randomly Occurred Nonlinearities.

1. Complex Networks

In the context of network theory, a complex network is a network (graph) with non-trivial topological features—features that do not occur in simple networks such as lattices or random graphs. The study of complex networks is a young and active area of scientific research inspired largely by the empirical study of real-world networks such as computer networks and social networks.

Many phenomena in nature can be modeled as a network, as brain structures, protein-protein interaction networks, social interactions and the Internet and WWW. All such systems can be represented in terms of nodes and edges indicating connections between nodes. In Internet, for example, the nodes represent routers and the edges the physical connections between them. In the same way, in transport networks, the nodes can represent the cities and the edges the highways that connect them. These edges can have weights, which can represent the flux of car in a highway or a frequency of interactions between two words in a language network.

An important characteristic of these networks is that they are not random, but have a more structured architecture. The topology of different networks, as protein-protein interaction networks and Internet, for example,

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are very close: they follow the power law, exhibiting a scale free structure. Finding the fundamental laws which generate these networks, modeling and characterizing them are the current challenges in complex network research.

2. Time Delay Concept and LMI

The time-delay phenomenon in spreading information through complex networks is well known to be ubiquitous in nature, technology, and society because of the finite speed of signal transmission over the links as well as the network traffic congestions. Hence, constant or time-varying discrete delays have been considered in many existing results about the synchronization problem for complex (neural) networks. It is worth mentioning that, as a particular kind of time delays, the continuously distributed time delays have also received much research attention since a network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths. Accordingly, the synchronization problem for continuous-time complex networks with discrete and/or distributed time delays has been extensively investigated.

2.1. *Linear Matrix Inequalities*

Linear Matrix Inequalities (LMIs) and LMI techniques have emerged as powerful design tools in areas ranging from control engineering to system identification and structural design. Three factors make LMI techniques appealing:
•A variety of design specifications and constraints can be expressed as LMIs.
•Once formulated in terms of LMIs, a problem can be solved exactly by efficient convex optimization algorithms (see LMI Solvers).
•While most problems with multiple constraints or objectives lack analytical solutions in terms of matrix equations, they often remain tractable in the LMI framework. This makes LMI-based design a valuable alternative to classical "analytical" methods.

2.1.1. *LMI Features*

Robust Control Toolbox LMI functionality serves two purposes:
•Provide state-of-the-art tools for the LMI-based analysis and design of robust control systems
•Offer a flexible and user-friendly environment to specify and solve general LMI problems (the LMI Lab)

Examples of LMI-based analysis and design tools include
•Functions to analyze the robust stability and performance of uncertain systems with varying parameters (popov, quadstab, quadperf ...)
•Functions to design robust control with a mix of H2, H∞, and pole placement objectives (h2hinfsyn)
•Functions for synthesizing robust gain-scheduled H∞ controllers (hinfgs)

For users interested in developing their own applications, the LMI Lab provides a general-purpose and fully programmable environment to specify and solve virtually any LMI problem. Note that the scope of this facility is by no means restricted to control-oriented applications.

2.2. *Randomly Occured Nonlinearity*

Motivated by the rich literature on complex network with switching structures, we aim to specifically address the randomly occurred nonlinearity (RON) that is an important phenomenon for the “blinking” networks discussed previously. As is well known, a wide class of practical systems are influenced by additive nonlinear disturbances that are caused by environmental circumstances. For complex networks with communication constraints, such nonlinear disturbances themselves may experience random abrupt changes, which may result from abrupt phenomena such as random failures and repairs of the components. Let us choose the networked systems and biological networks to justify the need of studying RONs. In a real-time networked environment, due to the limited bandwidth, network-induced packet losses, congestions, as well as quantization could be interpreted as a kind of external disturbances that occur in a probabilistic way and are randomly changeable in terms of their types and/or intensity. In a neural network, the signal transmission could be perturbed randomly from the release of probabilistic causes such as neurotransmitters. The randomly perturbed signals are in the form of spikes and most of the interaction with the other neurons takes place during the arrival of the spikes at the connection points, the synapses, which gives rise to a randomly switching interaction that is normally nonlinear. The so-called RONS, also called stochastic nonlinearities, have recently received some interest, and the filtering problem for discrete-time systems with stochastic nonlinearities has been thoroughly investigated. Nevertheless, to the best of the authors' knowledge, the synchronization problem for complex networks with specified RONs has not been fully investigated, and the purpose of this paper is therefore to shorten such a gap.

2.3 *Stochastic Delayed Discrete-time Complex Networks (SDDCN)*

The complex networks are often subject to noisy environment and, therefore, the stochastic modeling issue has been of vital importance in many branches of science such as neurotransmitters and network packet dropouts. In order to reflect more realistic dynamical behaviors, many researchers have recently investigated the problems of stochastic coupling and/or external stochastic disturbances for the synchronization of stochastic complex/neural networks. It should be pointed out that, another interesting random phenomenon, namely, randomly switching connections, has been paid considerable research attention in the literature. For example the global synchronization problem has been studied for continuous-time network with the so-called blinking connections that are randomly switched on and off with a given probability, and the frequency of switching is high compared to the network dynamics. The synchronization problem has been examined where the communication network topology changes randomly and is dictated by the agents' locations in the lattice. A stochastic genetic toggle switch model has been investigated where different time delays for transcription and translation and all reaction constant rates are randomly chosen from a range of values. it has been found that, for ensembles of yeast transcriptional network, those with deterministic Boolean rules are remarkably stable and those with random Boolean rules are only marginally stable. Motivated by the rich literature on complex network with switching structures, in this paper, we aim to specifically address the randomly occurred nonlinearity (RON) that is an important phenomenon for the "blinking" networks discussed previously. As is well known, a wide class of practical systems are influenced by additive nonlinear disturbances that are caused by environmental circumstances. For complex networks with communication constraints, such nonlinear disturbances themselves may experience random abrupt changes, which may result from abrupt phenomena such as random failures and repairs of the components. Let us choose the networked systems and biological networks to justify the need of studying RONs. In a real-time networked environment, due to the limited bandwidth, network-induced packet losses, congestions, as well as quantization could be interpreted as a kind of external disturbances that occur in a probabilistic way and are randomly changeable in terms of their types and/or intensity. In a neural network, the signal transmission could be perturbed randomly from the release of probabilistic causes such as neurotransmitters. The randomly perturbed signals are in the form of spikes and most of the interaction with the other neurons takes place during the arrival of the spikes at the connection points, the synapses, which gives rise to a randomly switching interaction that is normally nonlinear.

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2.3.1 *Creation of stochastic delayed discrete-time complex networks (SDDCN)*

A stochastic delayed discrete-time complex networks (SDDCN) is created with RONs, multiple stochastic disturbances, and mixed time delays. SDDCN constructed with four nodes where the state vector of each node is 2-D. Complex networks are widespread in real-world systems of engineering, physics, biology, and sociology. This thesis is concerned with the problem of synchronization for stochastic discrete-time drive-response networks. A dynamic feedback controller has been proposed to achieve the goal of the paper.

2.3.2 *Global Synchronization for SDDCN using LMI*

A novel matrix functional is constructed to attain new synchronization criteria, which are formulated in the form of linear matrix inequalities (LMIs). Then, based on the Lyapunov second method and LMI (linear matrix inequality) optimization approach, a delay-independent stability criterion is established that guarantees the asymptotical mean-square synchronization of two identical delayed networks with stochastic disturbances. The criterion is expressed in terms of LMIs, which can be easily solved by various convex optimization algorithms. Finally, two numerical examples are given to illustrate the proposed method

2.3.3 *Global Synchronization for SDDCN using combination of LMI and free-weighting matrix method*

If the delay is constant, one can transform a delayed system into a delay-free one by using state augmentation techniques. In this way, stability of such systems can be readily tested by employing classical results on stability analysis. Such an approach, however, is not always implementable as the dimension of the augmented system increases with the delay size. That is, when the delay is large, the augmented system will become much complex and thus difficult to analyze and synthesize. Moreover, the state augmentation technique is usually not applicable to the time-varying delay case, which is more frequently encountered than the constant delay case in practice. The reason is that for time-varying delay systems, the transformed systems usually have time-varying matrix coefficients, which are apparently difficult to analyze using available tools. Consequently, much effort has been made towards investigating the stability of discrete time-delay systems via Lyapunov approaches. However, it is worth mentioning that most of the results are concerned with the constant delay case, and according to the best of the authors' knowledge, little progress has been reported for the stability analysis of discrete- time systems with time-varying state delay, which motivates the present study. By introducing the "delay-fractioning" approach, we construct a novel matrix functional and utilize a combination of the free-weighting matrix method and the properties of Kronecker product. The utilization of the stochastic analysis technique results in the synchronization conditions expressed in terms of LMIs

2.3.4 *Global Synchronization for SDDCN using combination of optimized LMI and free-weighting matrix method*

An important feature of interior-point methods is that problem structure can be exploited to increase efficiency. The idea is very roughly as follows. In interior-point methods most of the computational effort is devoted to computing the Newton direction of a barrier or similar function. It turns out that this Newton direction can be expressed as the solution of a weighted least-squares problem of the same size as the original problem. Using conjugate-gradient and other related methods to solve these least-squares systems gives two advantages. First, by exploiting problem structure in the conjugate-gradient iterations, the computational effort required to solve the least- squares problems is much smaller than by standard direct" methods such as QR or Cholesky factorization. Second, it is possible to terminate the conjugate-gradient iterations before convergence, and still obtain an approximation of the Newton direction suitable for interior-point methods.

3. LMI and LMI Problem

A linear matrix inequality (LMI) is any constraint of the form

$$A(x) := A_0 + x_1 A_1 + \dots + x_N A_N < 0 \tag{1}$$

where

- •x = (x1, . . . , xN) is a vector of unknown scalars (the decision or optimization variables)
- •A0, . . . , AN are given symmetric matrices
- •< 0 stands for "negative definite," i.e., the largest eigenvalue of A(x) is negative

Note that the constraints A(x) > 0 and A(x) < B(x) are special cases of Equation 3-1 since they can be rewritten as – A(x) < 0 and A(x) – B(x) < 0, respectively.
The LMI of Equation 3-1 is a convex constraint on x since A(y) < 0 and A(z) < 0 imply that . As a result,

- •Its solution set, called the feasible set, is a convex subset of RN
- •Finding a solution x to Equation 3-1, if any, is a convex optimization problem.

Convexity has an important consequence: even though Equation 3-1 has no analytical solution in general, it can be solved numerically with guarantees of finding a solution when one exists. Note that a system of LMI constraints can be regarded as a single LMI since

$$\begin{cases} A_1(x) < 0 \\ \vdots \\ A_K(x) < 0 \end{cases} \text{ is equivalent to } A(x) := \operatorname{diag}(A_1(x), \ldots, A_K(x)) < 0$$

where diag (A1(x), . . . , AK(x)) denotes the block-diagonal matrix with A1(x), . . . , AK(x) on its diagonal. Hence multiple LMI constraints can be imposed on the vector of decision variables x without destroying convexity.

In most control applications, LMIs do not naturally arise in the canonical form of Equation 3-1 , but rather in the form
L(X1, . . . , Xn) < R(X1, . . . , Xn)where L(.) and R(.) are affine functions of some structured matrix variables X1, . . . , Xn. A simple example is the Lyapunov inequality

$$A^T X + XA < 0 \tag{2}$$

where the unknown X is a symmetric matrix. Defining x1, . . . , xN as the independent scalar entries of X, this LMI could be rewritten in the form of Equation 1. Yet it is more convenient and efficient to describe it in its natural form Equation 2, which is the approach taken in the LMI Lab.

4. Three Generic LMI Problems

Finding a solution x to the LMI system

$$A(x) < 0 \tag{3}$$

is called the feasibility problem. Minimizing a convex objective under LMI constraints is also a convex problem. In particular, the linear objective minimization problem.

$$\text{Minimize } c^T x \text{ subject to } A(x) < 0 \tag{4}$$

plays an important role in LMI-based design. Finally, the generalized eigenvalue minimization problem.

$$\text{Minimize } \lambda \text{ subject to } \begin{cases} A(x) < \lambda B(x) \\ B(x) > 0 \\ C(x) < 0 \end{cases} \tag{5}$$

is quasi-convex and can be solved by similar techniques. It owes its name to the fact that is related to the largest generalized eigenvalue of the pencil (A(x),B(x)).

Many control problems and design specifications have LMI formulations. This is especially true for Lyapunov-based analysis and design, but also for optimal LQG control, H∞ control, covariance control, etc. Further applications of LMIs arise in estimation, identification, optimal design, structural design, matrix scaling problems, and so on. The main strength of LMI formulations is the ability to combine various design constraints or objectives in a numerically tractable manner.

A nonexhaustive list of problems addressed by LMI techniques includes the following:

•Robust stability of systems with LTI uncertainty (μ-analysis)

•Robust stability in the face of sector-bounded nonlinearities (Popov criterion)

•Quadratic stability of differential inclusions

•Lyapunov stability of parameter-dependent systems

•Input/state/output properties of LTI systems (invariant ellipsoids, decay rate, etc.)

•Multi-model/multi-objective state feedback design

•Robust pole placement

•Optimal LQG control

•Robust H∞ control

•Multi-objective H∞ synthesis

•Design of robust gain-scheduled controllers

•Control of stochastic systems

•Weighted interpolation problems

To hint at the principles underlying LMI design, let's review the LMI formulations of a few typical design objectives.

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