PATTERN GENERATION FOR TWO-DIMENSIONAL CUTTING STOCK PROBLEM WITH LOCATION

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Abstract

Selection of feasible cutting patterns in order to minimize the raw material wastage which is known as cutting stock problem has become a key factor of the success in today's competitive manufacturing industries. In this paper, solving a rectangular shape medium size two-dimensional cutting stock problem is discussed. Our study is restricted to raw materials (main sheets) in a rectangular shape with fixed sizes, and cutting items are also considered as rectangular shape with known dimensions and location of each feasible cutting pattern inside the main sheet is given in Cartesian Coordinate Plane. The *Branch and Bound* approach in solving integer programming problems is used to solve the problem.

***Keywords*:** Two-dimensional cutting stock problem; Pattern generation; Branch and Bound Algorithm.

Introduction

Minimizing wastage is a key factor in improving productivity of a manufacturing plant. Wastage can occur in many ways and cutting stock problem can be described under the raw material wastage. Therefore, Operations Research plays a major role in minimizing raw material wastage or to maximizing space utilization of the raw material. Many people including scientists have conducted research to overcome above challenges. A cutting stock problem basically describes in two ways, One-Dimensional (1D) and Two-Dimensional (2D) cutting stock problems. An optimum cutting stock problem can be defined as cutting a main sheet into smaller pieces while minimizing total wastage of the raw material or maximizing overall profit obtained by cutting smaller pieces from the main sheet. Many researchers have worked on the cutting stock problem and developed different algorithms to solve the problem.

Gilmore et al (1961) conducted some of the earliest research in this area and one-dimensional cutting stock problem is solved using *Linear Programming Technique*. In this study, unlimited numbers of raw materials with different lengths are assumed available in stock, and a mathematical model has been developed to minimize the total cutting cost of the stock length of the feasible cutting patterns and *Column Generation Algorithm* has been developed to generate feasible cutting patterns. Then, Gilmore has claimed that feasible cutting patterns are increased with the required cutting items and *Linear Programming Technique* is not applicable to solved mathematical model with too many variables. Gilmore (1963) has made an approach for one-dimensional cutting stock problem as an extended of earliest paper (Gilmore et al (1961)) and cutting stock problem has been described as a NP-hard problem. A new and rapid algorithm for the knapsack problem and changes in the mathematical formulation[1] has been evolved and Gilmore has explained the procedure of the *Knapsack Method* using a test problem. Also, Saad (2001) has modified *Branch and Bound Algorithm* to find feasible cutting patterns for one-dimensional cutting stock problem and mathematical model has been developed to minimize the total cut loss. In the case study, Saad has selected four different types of steel coils to cut from the standard steel coil with the 130 cm length and width of the main coil and widths of the required coils are equal. *Branch and Bound Algorithm* has been explained using the example.

Other than to one-dimensional cutting stock problem, most of the researches have worked on the two-dimensional cutting stock problem. Among them, Christofides et al (1976) has presented a *Tree-Search Algorithm* for two-dimensional rectangular shaped cutting stock problem and a dynamic programming procedure for the solution of the unconstrained problem and a node evaluation method based on the transportation routine are used to produce upper bounds during the search. In this study, Christofides has used only guillotine cuts (a cut from one edge of the rectangle to the opposite edge which is parallel to the two remaining edges.) and the maximum number of pieces from each item can be produced as assumptions. The computational performance of the algorithm is illustrated using three test problems. Christofides has indicated that feasible cutting patterns are increased for the large number of items to be cut from the available main sheet and the *Tree-Search Algorithm* valid only up to medium size cutting stock problem.

Beasley (1985) has discussed the unconstrained two-dimensional cutting stock problem, with guillotine cuts and staged cuts (the cuts at the first stage are restricted to be guillotine cuts parallel to one axis, then the cuts at the second stage are restricted to be guillotine cuts parallel to the other axis and the cuts at the third stage are restricted to be guillotine cuts parallel to the original axis etc.). Beasley has presented both *Heuristic* and *Optimal Based Dynamic Programming* for staged cutting and guillotine cuttings. Computational results are presented for both staged cutting and general guillotine cutting. Beasley has indicated that the algorithm developed for staged cutting is more effective than the general guillotine cuts to generate feasible cutting patterns.

In this competitive manufacturing environment, there arise a problem to find an optimum layout of two-dimensional pieces with different shapes and sizes within the available main sheet of known dimensions. Hassan et al (2010) has made an approach to cut regular and irregular shaped pieces within rectangular main sheet of known dimensions and has transformed irregular shapes pieces into rectangles before the allocation is made. In the case study, Hassan has selected male trouser with size 42 cm. five types of pieces (two pieces of front side leg, two pieces of back side leg, belt, two pieces of side pockets and two pieces of back side pockets) have to be considered to tailor one trouser. For this task, imposed assumptions are the required pieces and main sheets should be in rectangular shape (if pieces are in irregular shapes it should be transformed to rectangular shape), the maximum length and width of each main sheet (fabric roll) are considered to be five meters and one meter respectively, all applied cuts are guillotine type and the cutting waste in each step cannot be more than the previous cutting step. Using *Simulated Annealing* process, single objective (minimizing the total cutting waste) two dimensional cutting stock problem has been solved.

Also, Chen et al (2011) has used a *hybrid algorithm* to solve two-dimensional cutting stock problem with irregular parts on multiple regular steel plates. Mathematical model has been developed to minimize cutting plane time and enhance steel use efficiency. In this study, first convert the irregular shape items into rectangular shapes and *Genetic Algorithm* has been applied to schedule parts layout sequence and finally apply *Improved Bottom Left Algorithm* to place the parts to a feasible location. Chen has described both *Genetic Algorithm* and *Improved Bottom Left Algorithm* using a case study.

There are different arrangements to cut required pieces from the existing raw material to maximize the used area and each arrangement is defined as a cutting pattern. Rodrigo et al (2012) has presented modified *Branch and Bound Algorithm* and a computer program using Matlab software package to generate feasible cutting patterns. Floor tile manufacturing plant has been selected to describe the modified *Branch and Bound Algorithm* as a case study.

In this study, modified *Branch and Bound Algorithm* is presented and a computer program using Matlab software package is developed to generate feasible cutting patterns and to define the location of each item in each pattern within the Cartesian coordinate plane for two-dimensional rectangular shape cutting stock problem as an extended of the earliest paper (Rodrigo et al (2012)).

Materials and Methods

Prior to finding minimum raw material wastage of two-dimensional cutting stock problem, rectangular shaped main sheet with known dimensions and required items are selected.
According to the selection, a mathematical model to minimize the wastage is formulated as follows:

Following notations are introduced to describe the model:

m = Number of items,
n = Number of patterns,
p_{ij} = Number of occurrences of the i^{th} item in the j^{th} pattern,
x_j = Number of main sheets being cut according to the j^{th} pattern,
c_j = Cutting loss for each j^{th} pattern,
d_i = Demand for the i^{th} item.

Mathematical Model:

$$\text{Minimize} \quad z = \sum_{j=1}^{n} c_j x_j \quad (\text{Total Cutting Loss})$$

$$\text{Subject to} \sum_{j=1}^{n} p_{ij} x_j \geq d_i \quad \text{for all } i = 1, 2, ..., m \quad (\text{Demand Constraints})$$

$$x_j, \; p_{ij} \geq 0 \text{ and integer for all } i, j,$$

The number of occurrences of the i^{th} piece in the j^{th} pattern (p_{ij}) needs to be determined to find the optimum solution (minimum-waste arrangement) for the given mathematical model. Therefore, modified *Branch and Bound Algorithm* is used to generate feasible cutting patterns.

Here, $\sum_{i=1}^{m} p_{ij} A_i \leq L \times W$ for all $j = 1, 2, ..., n$, where A_i, L and W are the area of the i^{th} item, length and width of the main sheet respectively.

***Modified Branch and Bound Algorithm*:**

Step 1: Arrange required lengths, l_i, i = 1, 2, …, m in decreasing order, ie $l_1 > l_2 > \ldots > l_m$,

where m = number of items.

Arrange required widths, w_i, i = 1, 2, …, m according to the corresponding length l_i, i = 1, 2, …, m.

Step 2: For i = 1, 2,…, m and j = 1 do Steps 3 to 6.

Step 3: set $a_{11} = \left[\left[L/l_1 \right]\right]$;

$$a_{ij} = \left[\left[\left(L - \sum_{z=1}^{i-1} a_{zj}\, l_z \right) \Big/ l_i \right]\right] \quad \text{—— (1),}$$

where L is the length of the main sheet.

Here, a_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern along the length of the main sheet and $[[y]]$ is the greatest integer less than or equal to y.

Step 4: If $a_{ij} > 0$, then set $b_{ij} = \left[\left[W/w_i \right]\right]$ ____(2)

else set $b_{ij} = 0$.

where W is the width of the main sheet.

Here, b_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern along the width of the main sheet.

Step 5: Set $p_{ij} = a_{ij} b_{ij}$,

where p_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern in the main sheet.

Step 6: Location of the i^{th} item in the j^{th} pattern and number of pieces from each item:

If $a_{ij} > 0$, then set

$$(x_i, y_i) = \left(\sum_{z=1}^{i} a_{zj} l_z - a_{ij} l_i \ , 0 \right), \left(\sum_{z=1}^{i} a_{zj} l_z \ , 0 \right), \left(\sum_{z=1}^{i} a_{zj} l_z \ , b_{ij} w_i \right), \left(\sum_{z=1}^{i} a_{zj} l_z - a_{ij} l_i \ , b_{ij} w_i \right);$$

$$N_i = a_{ij} \times b_{ij} ,$$

where N_i is the number of pieces can be cut from the i^{th} item in the j^{th} pattern along the length wise and width wise respectively and (x_i, y_i) are the coordinates of the each piece of i^{th} item in the j^{th} pattern within the main sheet.

Step 7: Cutting Loss

(i) Cut loss along the length of the main sheet:

$$c_u = \left(L - \sum_{i=1}^{m} a_{ij} l_i \right) \times W$$

For $i = 1, 2, ..., m$

If $\left(L - \sum_{i=1}^{m} a_{ij} l_i \right) \geq w_i$ and $W \geq l_i$, then

(Considering 90^o rotation for the given cutting items.)

$$\text{set } A_{ij} = \left\lfloor \left\lfloor \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \Big/ w_i \right\rfloor \right\rfloor ;$$

$$B_{ij} = \begin{cases} \left\lfloor \left\lfloor W / l_i \right\rfloor \right\rfloor , & \text{if } A_{ij} > 0 \\ 0 , & \text{otherwise.} \end{cases}$$

$$p_{ij} = p_{ij} + A_{ij}\, B_{ij}$$

$$\text{else set } A_{ij} = 0;$$

$$B_{ij} = 0;$$

$$P_{ij} = P_{ij}.$$

If $A_{ij} > 0$, then

$$\text{Set } (x_i, y_i) = \left(\left(\sum_{n=1}^{m} a_{nj}\, l_n \right) + \sum_{z=1}^{i} A_{zj}\, w_z - A_{ij}\, w_i \ , 0 \right), \left(\left(\sum_{n=1}^{m} a_{nj}\, l_n \right) + \sum_{z=1}^{i} A_{zj}\, w_z \ , 0 \right),$$

$$\left(\left(\sum_{n=1}^{m} a_{nj}\, l_n \right) + \sum_{z=1}^{i} A_{zj}\, w_z \ , B_{ij}\, l_i \right), \left(\left(\sum_{n=1}^{m} a_{nj}\, l_n \right) \sum_{z=1}^{i} A_{zj}\, w_z - A_{ij}\, w_i \ , B_{ij}\, l_i \right);$$

$$N_i = A_{ij} \times B_{ij} .$$

If $A_{ij} > 0$, then

$$\text{set } C_u = \left[\left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) - A_{ij}\, w_i \right] \times B_{ij}\, l_i ;$$

$$C_v = \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \times \left(W - B_{ij} l_i \right).$$

$$\text{else} \qquad C_u = \left(L - \sum_{i=1}^{m} a_{ij}\, l_i \right) \times W ,$$

where, A_{ij} and B_{ij} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the c_u rectangle respectively and C_u and C_v are the total cut loss area along the length and width of the main sheet respectively.

(ii) Cut loss along the width of the main sheet:

$c_v = (a_{ij}\, l_i) \times k_{ij}$.

Here, $k_{ij} = W - (b_{ij}\, w_i)$;

If $(b_{ij}\, w_i) = 0$, then

set $k_{ij} = 0$,

where k_{ij} is the remaining width of each item in each pattern

For $z \neq i$

If $(a_{ij}\, l_i) \geq l_z$ and $k_{ij} \geq w_z$, then

$$\text{set} \quad A_{zj} = \left[\left[\left({(a_{ij}\, l_i)}/{l_z} \right)\right]\right];$$

$$B_{zj} = \begin{cases} \left[\left[\left({k_{ij}}/{w_z} \right)\right]\right], & \text{if } A_{zj} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$p_{zj} = p_{zj} + A_{zj}\, B_{zj}$$

else set $A_{ij} = 0$;

$B_{ij} = 0$;

$P_{ij} = P_{ij}$.

If $A_{ij} > 0$, then

$$\text{Set } (x_i, y_i) = \left(\sum_{z=1}^{i} A_{zj}\, l_z - A_{ij}\, l_i \,,\, W - k_{ij} \right), \left(\sum_{z=1}^{i} A_{zj}\, l_z \,,\, W - k_{ij} \right), \left(\sum_{z=1}^{i} A_{zj}\, l_z \,,\, W - k_{ij} + b_{ij}\, w_i \right),$$

$$\left(\sum_{z=1}^{i} A_{zj}\, l_z - a_{ij}\, l_i \,,\, W - k_{ij} + b_{ij}\, w_i \right);$$

$N_i = (A_{ij} \times B_{ij})$.

If $A_{zj} > 0$, then

set $C_u = (a_{ij}\, l_i - A_{zj}\, l_z) \times B_{zj}\, w_z$;

$C_v = a_{ij}\, l_i \times (k_{ij} - B_{zj} l_z)$.

else $C_v = (a_{ij}\, l_i) \times k_{ij}$,

where, A_{zj} and B_{zj} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the c_v rectangle respectively.

Step 8: Set $r = m - 1$.

While $r > 0$, do Step 9.

Step 9: While $a_{rj} > 0$

set $j = j + 1$ and do Step 10.

Step 10: If $a_{rj} \geq b_{rj}$, then generate a new pattern according to the following conditions:

For $z = 1, 2, ..., r-1$

set $a_{zj} = a_{z\,j-1}$;

$b_{zj} = b_{z\,j-1}$.

For $z = r$

set $a_{z\,j} = a_{z\,j-1} - 1$;

if $a_{z\,j} > 0$, then set $b_{z\,j} = \left[\left[W / w_z \right]\right]$;

else set $b_{z\,j} = 0$.

For $z = r+1, \ldots, m$

calculate $a_{z\,j}$ and $b_{z\,j}$ using Equations (1) and (2).

For $i = 1, \ldots, m$

Set $p_{ij} = a_{ij} b_{ij}$.

Go to Step 6.

else generate a new pattern according to the following conditions:

For $z = 1, 2, ..., r-1$

set $a_{zj} = a_{z\,j-1}$;

$b_{zj} = b_{z\,j-1}$.

For $z = r$

set $a_{zj} = a_{z\,j-1}$;

$b_{zj} = b_{z\,j-1} - 1$.

For $z = r+1, \ldots, m$

calculate $a_{z\,j}$ and $b_{z\,j}$ using Equations (1) and (2).

For $i = 1, \ldots, m$

Set $p_{ij} = a_{ij} b_{ij}$.

Go to Step 6.

Step 11: Set $r = r - 1$.

Step 12: STOP.

Illustrative Example

Following example will illustrate how to generate feasible cutting patterns by minimizing total cutting waste:

A floor tile manufacturing plant uses rectangular shaped marble sheets of length 3000 mm and width 1400 mm as raw material to cut tiles according to the given specifications. The company has received an order for kitchen tiles according to the dimensions given below:

Table I
Required item dimensions and demand

Item number	**1**	**2**	**3**	**4**	**5**
Required dimensions (mm^2)	2200×600	1000×800	1400×600	1600×800	1000×600
Demand	1	5	4	1	3

Below illustrates the method described in the research paper to cut the main sheet according to the dimensions so that the total raw material wastage is minimized.

Results

Modified *Branch and Bound Algorithm* is applied to the above example to generate feasible cutting patterns as given below:

Step 1: For i = 1, 2, 3, 4, 5, 6, lengths l_i = 2200, 1600, 1400, 1000, 1000 and

w_i = 600, 800, 600, 800, 600

Length (L) and width (W) of the raw material are 3000 mm and 1400 mm respectively.

Dimensions of each item:

Item no (*i*)	**Length l_i (mm)**	**Width w_i (mm)**
1	2200	600
2	1600	800
3	1400	600
4	1000	800
5	1000	600

Step 2: For i = 1, 2, …, 5 and j = 1 do Steps 3 to 6.

Step 3: Set $a_{11} = \left[\left[L/l_1 \right]\right] = 1;$

$$a_{21} = \left[\left[\left[L - (l_1\, a_{11})\right] / l_2 \right]\right] = 0;$$

$$a_{31} = \left[\left[\left[L - (l_1\, a_{11}) - (l_2\, a_{21})\right] / l_3 \right]\right] = 0;$$

$$a_{41} = \left[\left[\left[L - (l_1\, a_{11}) - (l_2\, a_{21}) - (l_3\, a_{31})\right] / l_4 \right]\right] = 0;$$

$$a_{51} = \left[\left[\left[L - (l_1 a_{11}) - (l_2 a_{21}) - (l_3 a_{31}) - (l_4 a_{41})\right] \big/ l_5 \right]\right] = 0.$$

Step 4: $a_{11} > 0$, then set $b_{11} = \left[\left[W \big/ w_{11} \right]\right] = 2;$

$a_{21} = 0$, then $b_{21} = 0;$

$a_{31} = 0$, then $b_{31} = 0;$

$a_{41} = 0$, then $b_{41} = 0;$

$a_{51} = 0$, then $b_{51} = 0.$

Step 5:

$$\text{Set Pattern 1} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Step 6: $a_{11} > 0$, then set

$$(x_1, y_1) = \left(\sum_{z=1}^{1} a_{z1} l_z - a_{11} l_1 \,,\, 0\right), \left(\sum_{z=1}^{1} a_{z1} l_z \,,\, 0\right), \left(\sum_{z=1}^{1} a_{z1} l_z \,,\, b_{11} w_1\right), \left(\sum_{z=1}^{1} a_{z1} l_z - a_{11} l_1 \,,\, b_{11} w_1\right)$$
$$= (0,0), (2200,0), (2200,1200), (0,1200)$$

$$N_1 = a_{11} \times b_{11} = 1 \times 2 = 2$$

i.e. Two pieces can be cut from the Item 1 (2200 mm×600 mm) according to the above coordinates.

Step 7:

(i) Cutting loss along the length of the main sheet:

$$c_u = \left[L - (l_1 a_{11}) - (l_2 a_{21}) - (l_3 a_{31}) - (l_4 a_{41}) - (l_5 a_{51}) - (l_6 a_{61})\right] \times W$$
$$c_u = 800 \times 1400 = 1{,}120{,}000 \ \text{mm}^2.$$

For i = 1, 2

set $A_{ij} = 0;$

$B_{ij} = 0$. (Conditions are not satisfied given in Step 6 part (i))

For i = 3, dimensions of Item 3 are of length (l_3) 1400 mm and width (w_3) 600 mm and conditions are satisfied given in Step 6 part (i).

$$\text{set } A_{31} = \left[\left[{}^{800}\!/_{w_3} \right]\right] = 1;$$

$$B_{31} = \left[\left[{}^{1400}\!/_{l_3} \right]\right] = 1.$$

$A_{31} > 0$, then

$$\text{Set } (x_3, y_3) = \left(\left(\sum_{n=1}^{5} a_{n1}\, l_n\right) + \sum_{z=1}^{3} A_{z1}\, w_z - A_{31}\, w_3 \,,\, 0\right), \left(\left(\sum_{n=1}^{5} a_{n1}\, l_n\right) + \sum_{z=1}^{3} A_{z1}\, w_z \,,\, 0\right),$$

$$\left(\left(\sum_{n=1}^{5} a_{n1}\, l_n\right) + \sum_{z=1}^{3} A_{z1}\, w_z \,,\, B_{31}\, l_3\right), \left(\left(\sum_{n=1}^{5} a_{n1}\, l_n\right) \sum_{z=1}^{3} A_{z1}\, w_z - A_{31}\, w_3 \,,\, B_{31}\, l_3\right)$$

$$= (2200, 0),\ (2800, 0),\ (2800, 1400),\ (2200, 1400).$$

$$N_3 = A_{31} \times B_{31}$$
$$= 1 \times 1 = 1.$$

i.e. One piece can be cut from the Item 3 (1400 mm×600 mm) according to the above coordinates.

$$\text{set } C_u = [800 - (A_{31} w_3)] \times B_{31}\, l_3 = 200 \times 1400 = 280,000 \text{ mm}^2;$$

$$C_v = 800 \times [1400 - (B_{31} l_3)] = 800 \times 0 = 0 \text{ mm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

(ii) Cutting loss along the width of the main sheet:

$$c_v = a_{11} l_1 \times [1400 - (b_{11} w_1)] = 2200 \times 200 = 440,000 \text{ mm}^2$$

For $z = 2, \ldots, 6$

set $A_{ij} = 0$;

$B_{ij} = 0$. (Conditions are not satisfied given in Step 6 part (ii))

$$C_v = 2200 \times 200 = 440,000 \text{ mm}^2.$$

$$\text{Pattern 1} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$ and total cutting loss = 720, 000 mm^2.

Step 8: Set $r = 6 - 1 = 5 > 0$

Step 9: $a_{51} = 0$, then go to Step 11.

Step 11: Set $r = 5 - 1 = 4 > 0$

Step 9: $a_{41} = 0$, then go to Step 11.

Step 11: Set $r = 4 - 1 = 3 > 0$

Step 9: $a_{31} = 0$, then go to Step 11.

Step 11: Set $r = 3 - 1 = 2 > 0$

Step 9: $a_{21} = 0$, then go to Step 11.

Step 11: Set $r = 2 - 1 = 1 > 0$

Step 9: $a_{11} > 0$, then

set $j = j + 1 = 2$ and go to Step 10.

Step 10: $a_{11} < b_{11}$, then generate a new pattern j (= 2) according to the following conditions:

$$\text{set } a_{12} = a_{11} = 1 ; b_{12} = b_{11} - 1 = 1.$$

$$a_{22} = \left[\left[\left[L - (l_1 a_{12})\right] / l_2 \right]\right] = 0 ; b_{22} = 0.$$

$$a_{32} = \left[\left[\left[L - (l_1 a_{12}) - (l_2 a_{22})\right] / l_3 \right]\right] = 0 ; b_{32} = 0.$$

$$a_{42} = \left[\left[\left[L - (l_1 a_{12}) - (l_2 a_{22}) - (l_3 a_{32})\right] / l_4 \right]\right] = 0 ; b_{42} = 0.$$

$$a_{52} = \left[\left[\left[L - (l_1 a_{12}) - (l_2 a_{22}) - (l_3 a_{32}) - (l_4 a_{42})\right] / l_5 \right]\right] = 0 ; b_{52} = 0.$$

$$\text{Set Pattern 2 } = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Step 6: $a_{12} > 0$, then set

$$(x_1, y_1) = \left(\sum_{z=1}^{1} a_{z2}\, l_z - a_{12}\, l_1 \, , \, 0 \right), \left(\sum_{z=1}^{1} a_{z2}\, l_z \, , \, 0 \right), \left(\sum_{z=1}^{1} a_{z2}\, l_z \, , \, b_{12}\, w_1 \right), \left(\sum_{z=1}^{1} a_{z2}\, l_z - a_{12}\, l_1 \, , \, b_{12}\, w_1 \right)$$
$$= (0,0), (2200,0), (2200,600), (0,600)$$

$$N_1 = a_{12} \times b_{12} = 1 \times 1 = 1.$$

i.e. one piece can be cut from the Item 1 (2200 mm×600 mm) according to the above coordinates.

Step 7:

(i) Cutting loss along the length of the main sheet:

$$c_u = [L - (l_1 a_{12}) - (l_2 a_{22}) - (l_3 a_{32}) - (l_4 a_{42}) - (l_5 a_{52}) - (l_6 a_{62})] \times W$$
$$c_u = 800 \times 1400 = 1{,}120{,}000 \text{ mm}^2.$$

For $i = 1, 2$

set $A_{ij} = 0;$

$B_{ij} = 0.$

For $i = 3$, dimensions of Item 3 are of length (l_3) 1400 mm and width (w_3) 600 mm and conditions are satisfied given in Step 6 part (i).

$$\text{set } A_{32} = \left[\left[800 / w_3 \right]\right] = 1;$$

$$B_{32} = \left[\left[1400 / l_3 \right]\right] = 1.$$

$A_{32} > 0$, then

$$\text{Set } (x_3, y_3) = \left(\left(\sum_{n=1}^{5} a_{n2}\, l_n\right) + \sum_{z=1}^{3} A_{z2}\, w_z - A_{32}\, w_3 ,\ 0\right), \left(\left(\sum_{n=1}^{5} a_{n2}\, l_n\right) + \sum_{z=1}^{3} A_{z2}\, w_z ,\ 0\right),$$

$$\left(\left(\sum_{n=1}^{5} a_{n2}\, l_n\right) + \sum_{z=1}^{3} A_{z2}\, w_z ,\ B_{32}\, l_3\right), \left(\left(\sum_{n=1}^{5} a_{n2}\, l_n\right) \sum_{z=1}^{3} A_{z2}\, w_z - A_{32}\, w_3 ,\ B_{32}\, l_3\right)$$

$$= (2200, 0),\ (2800, 0),\ (2800, 1400),\ (2200, 1400).$$

$$N_3 = A_{32} \times B_{32} = 1 \times 1 = 1.$$

i.e. One piece can be cut from the Item 3 (1400 mm×600 mm) according to the above coordinates.

$$\text{set } C_u = [800 - (A_{32} w_3)] \times B_{32}\, l_3 = 200 \times 1400 = 280{,}000 \text{ mm}^2;$$

$$C_v = 800 \times [1400 - (B_{32} l_3)] = 800 \times 0 = 0 \text{ mm}^2.$$

$$\text{Pattern 1} = \text{Pattern 1} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

(ii) Cutting loss along the width of the main sheet:

$$c_v = a_{12} l_1 \times [1400 - (b_{12} w_1)] = 2200 \times 800 = 1,760,000 \text{ mm}^2.$$

For z = 2, dimensions of Item 2 are of length (l_2) 1600 mm and width (w_2) 800 mm and conditions are satisfied given in Step 6 part (ii).

$$\text{set } A_{22} = \left[\left[2200/l_2 \right]\right] = 1;$$

$$B_{22} = \left[\left[800/w_2 \right]\right] = 1.$$

If $A_{22} > 0$, then

$$\text{Set } (x_2, y_2) = \left(\sum_{z=1}^{2} A_{z2} l_z - A_{22} l_2 , W - k_{22} \right), \left(\sum_{z=1}^{2} A_{z2} l_z , W - k_{22} \right), \left(\sum_{z=1}^{2} A_{z2} l_z , W - k_{22} + b_{22} w_2 \right),$$
$$\left(\sum_{z=1}^{2} A_{z2} l_z - A_{22} l_2 , W - k_{22} + b_{22} w_2 \right)$$
$$= (0, 600), (1600, 600), (1600, 1400), (0, 1400).$$

$$N_2 = A_{22} \times B_{22} = 1 \times 1 = 1.$$

i.e. One piece can be cut from the Item 2 (1600 mm×800 mm) according to the above coordinates.

$$C_u = [2200 - (A_{22} w_2)] \times B_{22} l_2 = 600 \times 800 = 480,000 \text{ mm}^2,$$

$$C_v = 2200 \times [800 - (B_{22} l_2)] = 2200 \times 0 = 0 \text{ mm}^2.$$

$$\text{Pattern 2} = \text{Pattern 2} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\text{Pattern 2} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and total cutting loss} = 760{,}000 \text{ mm}^2.$$

The algorithm proceeds in the same manner to generate all the cutting patterns shown in Table II for the $3000 \text{ mm} \times 1400 \text{ mm}$ standard dimension.

The Table II exhibits the generated cutting patterns for optimum waste:

Table II

Generated cutting patterns

Required dimensions	Cutting patterns																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2200×600 mm^2	2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
1600×800 mm^2	0	1	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1400×600 mm^2	1	1	2	1	1	3	2	1	1	0	4	3	3	2	2	0	0	1	0	0
1000×800 mm^2	0	0	0	2	0	0	0	1	0	1	0	1	0	2	1	3	3	2	1	0
1000×600 mm^2	0	0	0	0	2	0	1	1	2	2	0	1	2	1	2	0	3	2	5	6
Cut loss ($\times 10^4$) mm^2	72	76	120	44	84	40	64	68	88	92	84	28	48	32	52	48	0	56	40	60

There are 20 feasible cutting patterns available to cut raw material with the dimensions $3000 \text{ mm} \times 1400 \text{ mm}$ into required rectangular shaped items. The mathematical model is developed to design generated cutting patterns so that waste (cut loss) will be minimized and the optimum solution to the model is given in Table III:

Table III
Optimum solution

Required Dimensions	Optimal Patterns			Demand
	4	6	17	
2200×600 mm^2	1	0	0	1
1600×800 mm^2	0	1	0	1
1400×600 mm^2	1	3	0	4
1000×800 mm^2	2	0	3	5
1000×600 mm^2	0	0	3	3
# of sheets from each pattern	1	1	1	

$Z_{min} = 840,000$ mm^2 (Total cut loss = 840, 000 mm^2).

Location of the 4th optimal pattern:

Item 1 – $(0,0)$, $(2200,0)$, $(2200,600)$, $(0,600)$;

Item 2 – No pieces;

Item 3 – $(2200,0)$, $(2800,0)$, $(2800,1400)$, $(2200,1400)$;

Item 4 – $(0,600)$, $(2000,600)$, $(2000,1400)$, $(0,1400)$;

Item 5 – No pieces.

Conclusion

In this study, a cutting stock problem is formulated as a mathematical model based on the concept of cutting patterns. As given in Table II, twenty cutting patterns are generated and only three cutting patterns are selected as given in Table III to cut the main sheet according to the requirements. In each cutting pattern, locations of each piece within the main sheet are given using the modified *Branch and Bound Algorithm*. In this case study, the plant assumes that all the extra pieces from each item as wastage. Also, dimensions of cutting items are large and the total cut loss can be decreased if there are smaller rectangular shaped cutting items.

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