0-EDGE MAGIC LABELING FOR SOME CLASS OF GRAPHS

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Abstract:
In this paper we introduce new labeling called 0-Edge Magic Labeling, and also shown the existence of this Labeling for Some Class of graphs.

Keywords: Graph labeling, 0-Edge Magic Labeling.

1. Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs are becoming an increasingly useful family of Mathematical Models from abroad range of applications. The graph labeling problem that appears in graph theory has a fast development recently. This problem was first introduced by Alex Rosa in 1967. Since Rosa's article, many different types of graph labeling problems have been defined around this. This is not only due to its mathematical importance but also because of the wide range of the applications arising from this area, for instance, x-rays, crystallography, coding theory, radar, astronomy, circuit design, and design of good Radar Type Codes, Missile Guidance Codes and Convolution Codes with optimal autocorrelation properties and communication design. An enormous body of literature has grown around the subject in about 1300 papers. They gave birth to families of graphs with attractive names such as graceful, Harmonious, felicitous, elegant, cordial, magic antimagic, bimagic and prime labeling etc. A useful survey to know about the numerous graph labeling methods is the one by J.A. Gallian recently [2011].All graphs considered here are finite simple and undirected.

2. Preliminaries

Definition 2.1. The vertex-weight of a vertex v in G under an edge labeling is the sum of edge labels corresponding to all edges incident with v. Under a total labeling, vertex-weight of v is defined as the sum of the label of v and the edge labels corresponding to the entire edges incident with v. If all vertices in G have the same weight k, we call the labeling vertex-magic edge labeling or Vertex-magic total labeling respectively and we call k a magic constant. If all vertices in G have different weights, then the labeling is called vertex-antimagic edge labeling or vertex-antimagic total labeling.

Definition 2.2. The edge-weight of an edge e under a vertex labeling is defined as the sum of the vertex labels corresponding to every vertex incident with e under a total labeling, we also add the label of e. Using edge-weight, we derive edge-magic vertex or edge-magic total labeling and edge-antimagic vertex or edge-antimagic total labeling

Definition 2.3. A (p, q) graph G is said to be $(1,0)$ edge-magic with the common edge count k if there exists a bijection $f : V(G) \rightarrow \{1, \ldots, p\}$ such that for all $e = uv \in E(G), f(u) + f(v) = k$. It is said to be $(1, 0)$ edge-antimagic if for all $e = (u,v) \in E(G), f(u) + f(v)$are distinct.

Definition 2.4. A (p,q) graph G is said to be $(0,1)$ vertex-magic with the common vertex count k if there exists a bijection $f: E(G) \rightarrow \{1, \ldots, q\}$ such that for each $u \in V(G), e \in\Sigma \quad f(e) =k$ for all $e = uv \in E(G)$ with $v \in V(G).$

It is said to be *(0, 1)* vertex-antimagic if for each $u \in V(G)$, $e \in \Sigma$ $f(e)$ are distinct for all $e = uv \in E(G)$ with $v \in V(G)$.

Definition 2.5. A *(p,q)* graph *G* is said to be *(1,1)* edge-magic with the common edge count *k* if there exists a bijection $f : V(G) \cup E(G) \rightarrow \{1, \ldots, p+q\}$ such that $f(u) + f(v) + f(e) = k$ for all $e = uv \in E(G)$. It is said to be *(1,1)* edge-antimagic if $f(u) + f(v) + f(e)$ are distinct for all $e = uv \in E(G)$.

3. Main Result

0-Edge Magic Labeling: Let G = (V, E) be a graph where $V = \{v_i, 1 \le i \le n\}$, and $E = \{v_iv_{i+1}, 1 \le i \le n\}$. Let f : V→ {-1, 1}, and f^* : E → {0}, such that all uv ∈ E, $f^*(uv) = f(u) + f(v) = 0$ then the labeling is said to be 0-Edge Magic labeling.

Theorem 3.1 P_n admits 0-edge magic labeling.
Proof: Let G = (V, E) be a graph where $V = \{v_i, 1 \le i \le n\}$ and $E = \{v_iv_{i+1}, 1 \le i \le n\}$.
Let f: V→ {-1, 1}
Such that $f(v_i) = (-1)^i$, $1 \le i \le n$,
$f^*(v_i v_{i+1}) = (-1)^i + (-1)^{i+1}$
$= -1+1 = 0$.if i is odd
Also $f^*(v_i v_{i+1}) = (-1)^i + (-1)^{i+1}$
$= 1-1 = 0$,if i is even
Hence the theorem.

Theorem 3.2 C_n admits 0-edge magic labeling.
Proof: Let G = (V, E) be a graph where $V = \{v_i, 1 \le i \le n\}$ and $E = \{v_iv_{i+1}, 1 \le i \le n\}$.
Let f: V→ {-1, 1}
Such that $f(v_i) = (-1)^i$,
$1 \le i \le n$, $f^*(v_i v_{i+1}) = (-1)^i + (-1)^{i+1}$
$= -1+1 = 0$.if i is odd
Also $f^*(v_i v_{i+1}) = (-1)^i + (-1)^{i+1}$
$= 1-1 = 0$.if i is even.
Hence the theorem.

Theorem 3. 3 C_n^+ admits 0- edge magic labeling.
Proof: Let G = (V, E) be a graph where $V = \{v_i, 1 \le i \le n\}$ and $E = \{v_iv_{i+1}, 1 \le i \le n\}$.
Let f: V→ {-1, 1}
Such that $f(v_i) = (-1)^i$, $1 \le i \le n$,
$f^*(v_i v_{i+1}) = (-1)^i + (-1)^{i+1}$ $= -1+1 = 0$.if i is odd
Also $f^*(v_i v_{i+1}) = (-1)^i + (-1)^{i+1}$
$= 1-1 = 0$.if i is even
C_n^+ is obtained by adding one pendent edge to each vertex, which leads to 2n vertices let v_j: $1 \le j \le n$ be the incident vertices to each v_i.
Let $v_j = -1$ if $v_i = 1$or $v_j = 1$ if $v_i = -1$.
Clearly $f^*(v_i v_j) = (-1)^i + (-1)^j$
$= -1+1 = 0$.if i is odd
Also $f^*(v_i v_j) = (-1)^i + (-1)^j$
$= 1-1 = 0$.if i is even.
Hence C_n^+ admits zero sum labeling.

Theorem 3.4. If G admits 0- edge magic then G^+ admits 0- edge magic.
Proof: Let G = (V, E) be a graph where $V = \{v_i, 1 \le i \le n\}$ and $E = \{v_iv_{i+1}, 1 \le i \le n\}$.
Let f: V→ {-1, 1}, and f^*:E→{0}, such that all $v_i v_{i+1} \in E$ $f^*(v_i v_{i+1}) = f(v_i) + f(v_{i+1}) = 0$.
Let $G^+ = (V, E) \cup \{v_j, 1 \le j \le n\} \cup \{v_iv_j, 1 \le i, j \le n\}$, then let g : V→ {-1, 1}, and g^* : E → {0}, then for all v_i v_j, ∈ E ,
Let $v_j = -1$ if $v_i = 1$or $v_j = 1$ if $v_i = -1$,
Then $g^*(v_i v_j) = g^*(v_i) + g^*(v_j) = 0$.
Hence the proof.

Corollary : m copies of C_n, $nP_2UmC_{n,}$ $nP_2UmK_{1,}$ admits 0- edge magic labeling, but Ladder ,Wheel graph, Windmill graph, K_n admits 0- edge magic labeling if $n\equiv0(mod2)$.

Conclusion

In this paper we have investigated some Class of graphs admitting 0-edge magic labeling.

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