

Discovering Non-Redundant Association Rules using MinMax Approximation Rules

R. Vijaya Prakash
Department Of Informatics
Kakatiya University,
Warangal, India
vijprak@hotmail.com

Dr.A. Govardhan
Department. Of Comp. Sci. & Eng.
JNT University,
Hyderabad, India
govardhan_cse@yahoo.co.in

Prof. SSVN. Sarma
Dept. Of Comp. Sci. & Eng.
Vaagdevi college of Eng.
Warangal, India
ssvn.sarma@gmail.com

Abstract

Frequent pattern mining is an important area of data mining used to generate the Association Rules. The extracted Frequent Patterns quality is a big concern, as it generates huge sets of rules and many of them are redundant. Mining Non-Redundant Frequent patterns is a big concern in the area of Association rule mining. In this paper we proposed a method to eliminate the redundant Frequent patterns using MinMax rule approach, to generate the quality Association Rules.

Keywords—Frequent Pattern, Association Rule Mining (ARM), Non-Redundant Frequent Pattern, MinMaxExact, MinMaxApproax Rules

1. Introduction

Frequent pattern mining is an important area of Data mining research. The problem of mining Frequent patterns was first introduced in Agrawal et al.'s pioneering work [1] for market basket analysis in the form of Association Rule mining. Mining Frequent patterns has become an important data mining problem and has received considerable attention in recent years. The goal is to discover frequently occurring patterns from data of various forms.

The Association Rule of the form $A \rightarrow B$ holds the transaction set D with support s , where s is percentage of transaction in D that contain $A \cup B$, this is the probability of $A \cup B$ i.e. $P(A \cup B)$. The rule $A \rightarrow B$ has confidence c in the transaction set D , where c is the percentage of transaction in D containing A that also contains B . This is the conditional probability $P(B/A)$ [1]. The Association Rules that satisfy a user specified thresholds called minimum support or minsupp , and minimum confidence or minconf are called Frequent patterns.

2. Related Work

Researchers have proposed various strategies to Non-Redundant Frequent Itemset Patterns. This has led to research on problems such as mining of *Closed Itemsets* [2], *maximal Itemsets* [6], *non-derivable Itemsets* [3], and more recently *pattern profiles* [4].

Closed Itemsets and non-derivable itemsets are lossless forms of compressing Frequent Itemset patterns, i.e. the full list of Frequent Itemsets and associated frequency counts can be exactly derived from the compressed representation. It is not clear which compressing scheme is better.

Researchers have found that for some datasets and support thresholds we have $|NDI| < |Closed|$, while other datasets and support thresholds have $|Closed| < |NDI|$ [3]. Maximal Itemsets allow greater compression when compared with Closed Itemsets, but the representation is lossy – the list of Frequent Itemsets can be exactly

computed but the exact frequency counts associated with these Frequent Itemsets cannot be determined. There are some other lossy representations besides maximal itemsets. Top-k patterns approach by Han et al. [8] presents the most Frequent k Closed Itemsets to the end-user. Xin et al. [9] extend this work to extract *redundancy aware* top-k Frequent Itemset patterns. *Error-tolerant patterns* by Yang et al. [5] and Pei et al. [10] allow certain amount of fluctuations in evaluating supports of itemset patterns. An approach by Afrati et al. [7] uses K Itemsets to recover a collection of Frequent Itemsets. However, it is not clear how to recover the support information with their approach.

Yan et al. [4] demonstrate that the pattern profile approach can effectively summarize itemsets for Non-Redundant Frequent Itemsets, resulting in good compression while retaining high recovery accuracy. However, from an efficiency perspective it is not clear how well this approach will scale to large datasets. Unfortunately, there is no clear way to alleviate this problem. The pattern profiles are not itemset patterns themselves. It's not clear how to use them for data analysis.

Zaki's [15] approaches are based on frequent closed itemsets using a FCA (formal concept analysis) framework, Zaki [15] make use of the closure of the Galois connection to extract non-redundant rules from frequent closed itemsets instead of from frequent itemsets. The approach proposed by Zaki. [15] extracts rules; called the most general rules; that have the shortest antecedent and shortest consequent in an equivalent class of rules with the same confidence and the same support. All other rules in the equivalent class are considered redundant to the extracted rules. The extracted rules (i.e., the most general rules) constitute a generating rule set from which all other rules can be derived. However, the generating set may not retain the same inference capacity as the entire rule set.

3. Research Methodology

3.1 Association Rules

Let $I = \{I_1, I_2, \dots, I_m\}$ be a set of unique items with m members, t is a transaction that contains a set of items from I so that $t \subseteq I$ and T is a database or dataset containing a set of identifiable transactions t . As mentioned an association rule is implication of the following form $X \rightarrow Y$, where $X, Y \subseteq I$, and are known as itemsets, containing a set of items where $X \cap Y = \emptyset$. The closure operation of the Galois connection provides the definition for closed Itemsets [11]. The definition is as follows; $i \in I$ and $t \in T$, if an item i appear in a transaction t , then both i and t have a binary relation denoted by iRt . The following mappings define the Galois connection of the binary relation, where $X \subseteq I, Y \subseteq T$

$$\tau : 2^I \rightarrow 2^T, \tau(X) = \{t \in T \mid \forall i \in X, iRt\}$$

$$\gamma : 2^T \rightarrow 2^I, \gamma(Y) = \{i \in I \mid \forall t \in Y, iRt\}$$

From this $\tau(X)$ known as the transaction relation of X , while $\gamma(Y)$ is known as the item relation of Y

Definition 3.1 (Support): The support of an Itemset X , which can be denoted as $supp(X)$, is determined as the percentage of the transactions which contain X . Therefore

$$supp(X) = |\tau(X)|/|T|$$

Definition 3.2 - Confidence: The confidence of an association rule of the form $X \rightarrow Y$ can be denoted $conf(X \rightarrow Y)$ and is determined as the percentage of the transactions which contain $X \cup Y$ out of the transactions which only contain X . Therefore

$$conf(X \rightarrow Y) = |\tau(X \cup Y)| / |\tau(X)|.$$

We can also formally define two more concepts, which are important when it comes to Non-Redundant association rule mining; Closed Itemset and Generator; as follows:

Definition 3.3 (Closed Itemset): If we let X be a subset of I then X is a closed Itemset if and only if the following holds, $\gamma\tau(X) = X$.

Definition 3.4 (Generator): An Itemset $g \in 2^I$ is considered to be a generator of a closed Itemset $c \in 2^I$ if and only if the following holds, $c = \gamma\tau(g)$ and $g \subset \gamma\tau(g)$. If there is no $g' \subset g$ such that $\gamma\tau(g') = c$.

Usually there are two sub problems that association rule mining can be decomposed into; finding Frequent Itemsets which have a support greater than or equal to a predefined minimum support and using those Frequent Itemsets to generate Association Rules which satisfy both the minimum support and minimum confidence thresholds.

3.2 Redundancy Definition.

Looking at Table IV, the rules contained within are considered to be useful based on the fact their support and confidence values meet or exceed a predefined minimum support and minimum confidence. However, some of these rules do not contain or present new information to a user. In particular, the consequent concluded by some rules can be obtained from other rules with the same or higher confidence level but without requiring more conditions to be satisfied. For example, we can obtain the rule $A \rightarrow B$ by transitivity from the two rules $A \rightarrow E$, and $E \rightarrow B$. The rule $CE \rightarrow B$ can be obtained by augmentation of the two rules $E \rightarrow B$ and $C \rightarrow B$, etc. We can see that the redundant rules have an antecedent of equal or greater length and a consequent of equal or shorter length respectively, while the confidence of the redundant is not greater than the corresponding Non-Redundant rules. From this the following definition defines this kind of redundant rules.

Definition:3.5 (Redundant Rules): If we let $X \rightarrow Y$ and $X' \rightarrow Y'$ be two Association Rules with confidences cf and cf' respectively, then $X \rightarrow Y$ is said to be a redundant rule to $X' \rightarrow Y'$ if $X' \subseteq X$, $Y' \subseteq Y$ and $cf \leq cf'$.

From definition 3.1 if we have an association rule $X \rightarrow Y$, if there is no other rule $X' \rightarrow Y'$ in existence such that the confidence of $X' \rightarrow Y'$ is equal to or larger than the confidence of $X \rightarrow Y$ and $X' \subseteq X$, $Y' \subseteq Y$, then the association rule $X \rightarrow Y$ is said to be Non-Redundant.

Eliminating redundant Association Rules safely without damaging the capacity of the remaining rules is essential and it is crucial to successfully define a boundary between redundant and no redundant in order to ensure safe redundancy removal. Several different approaches to achieve this have been proposed [13][14][15]. However none have specifically discussed the boundary. In This paper, we proposed to use the Certainty Factor (CF) to determine the boundary. If deleting an association rule does not reduce the CF value of the remaining Association Rules then the deletion of that rule is considered to be safe.

The concept of the certainty factor was first proposed in [17] in order to express the level of accuracy and truth behind an association rule and also determine how reliable the antecedent of the given rule is. Certainty factor is founded on two functions; the measure of belief $\delta(X, Y)$ and the measure of disbelief $\gamma(X, Y)$ for a rule of the form $X \rightarrow Y$. The functions of δ and γ are given as follows:

$$\delta(X, Y) = \begin{cases} \frac{p(Y/X) - p(Y)}{p(Y)}, & \text{if } p(Y/X) > P(Y) \\ 0, & \text{otherwise} \end{cases}$$

$$\gamma(X, Y) = \begin{cases} \frac{p(Y/X) - p(Y)}{1 - p(Y)}, & \text{if } p(Y/X) \leq P(Y) \\ 0, & \text{otherwise} \end{cases}$$

The $P(Y/X)$ and $P(Y)$ represent the confidence of the Association rule and the support of the consequent respectively. For both δ and γ the values range between 0 and 1 and measure the strength of the belief or disbelief in the consequent Y given the antecedent X . Thus, $\delta(X, Y)$ weighs how much the antecedent X increases the possibility of consequent Y occurring, while $\gamma(X, Y)$ weighs how much the antecedent X decreases the possibility of consequent Y occurring. If $P(Y/X)$ equals 1, then the antecedent completely supports the consequent and thus $\delta(X, Y)$ will be 1. On the other hand, if $P(Y/X)$ is equal to 0, then this indicates that the antecedent completely denies the consequent and thus $\gamma(X, Y)$ will be 1. The total strength of the belief or disbelief captured by the association rule is measured by the certainty factor, which is defined as follows:

$$CF(X, Y) = \delta(X, Y) - \gamma(X, Y)$$

The value of the certainty factor will be between 1 and -1, where negative values represent the cases where the antecedent is against (denying) the consequent. Positive values indicate that the antecedent supports the consequent. The certainty factor value of 0 means that the antecedent does not influence the consequent. Association Rules with a high certainty factor value are the most useful as they represent strong positive associations between the rule's antecedent and consequent. The aim of association rule mining is to discover these rules that have strong positive associations. It is therefore proposed that the certainty factor can be used to measure the strength of discovered Association Rules

The following Definitions (3.6) states that the certainty factor value of a redundant rule, as defined by Definition 3.5 will never be greater than the certainty factor of the corresponding Non- Redundant rules. It thus means that the association between the antecedent and consequent of the Non-Redundant rule is stronger than any corresponding redundant rule.

Definitions 3.6: Let $X \rightarrow Y$ and $X' \rightarrow Y'$ represent two Association Rules. If $Y' \subseteq Y$ and $P(Y/X) \geq P(Y'/X')$ then $CF(X, Y) \geq CF(X', Y')$.

3.3 Concise and Lossless Representation

The development of a concise and lossless representation is a promising way for improving the quality of discovered associations. Some work has previously been done in this area [12][14][17][18], with work by [16] proposing two condensed bases which represent Non-Redundant Association Rules. These bases are defined as follows:

Definition 3.7 (MinMax Exact Basis): Let C be the set of Frequent Closed Itemsets and for each Frequent Closed Itemset c , let G_c be the set of minimal generators for c . The MinMax exact basis thus is [19][20]:

$$\text{MinMaxExact} = \{r: g \rightarrow (c \setminus g) \mid c \in C \text{ and } g \in G_c \text{ and } g \neq c\}$$

Definition 3.8 (MinMax Approximate Basis): Let C be the set of Frequent Closed Itemsets and let G be the set of minimal generators of the set of Frequent Closed Itemsets in C . The MinMax approximate basis is [19][20]:

$$\text{MinMaxApprox} = \{r: g \rightarrow (c \setminus g) \mid c \in C \text{ and } g \in G_c \text{ and } \gamma \circ \tau(g) \subset C\}$$

Rules which have a confidence of less than 1 are known as approximate rules, while those with a confidence equal to 1 are known as exact rules.

C. Non-Redundant Association Rule Mining Algorithms

The generic representation that results from a coupling of the MinMaxExact Basis with the MinMaxApproximate Basis results in a more concise set of Association Rules which are Non-Redundant, and lossless. Here we present the algorithms for Non-Redundant association rule mining in single level datasets.

Input : Set of Frequent closed itemsets and generators

Output: Set of non – redundant exact basis rules

1. exactrules $\leftarrow \emptyset$
2. for all $c \in C$
3. for all $g \in G_c$
if $\forall c' \in C \mid c' \subset c$ and $\forall g' \in G_c$
then exactRules := exactRules $\cup \{g \rightarrow c\}$
4. end
5. end
6. return exactRules

Input : Set of Frequent Closed Itemsets and generators

Output: Set of non – redundant approximate basis rules

1. approxRules $\leftarrow \emptyset$
2. for all $c \in C$
3. for all $g \in G_c$ such that $c \supset \gamma \circ \tau(g)$
4. if $\forall c' \in C$ and $\forall g' \in C \mid c' \supset \gamma \circ \tau(g')$ and $g' \subseteq g$
or $\text{conf}(g \rightarrow c) > \text{conf}(g' \rightarrow c')$
5. then approxRules := approxRules $\cup \{g \rightarrow c\}$
6. end
7. end
8. return approxRules

4. Experiments

To explain the concept mentioned above we consider an example data base, as shown in Table I There are six transactions in the database with their transaction identifiers (TID's) ranging from 1 to 6. The universal itemset $I = \{A, B, C, D, E\}$, where A, B, C, D and E can be any items in the supermarket. For instance, A = ‘‘bread’’, B = ‘‘milk’’, C = ‘‘sugar’’, D = ‘‘coffee’’, and E = ‘‘biscuit’’.

TABLE I : Transaction Database

TID	Items Bought
1	ABDE
2	BCE
3	ABDE
4	ABCE
5	ABCDE
6	BCD

There are totally 25(=32) itemsets. {A}, {B}, {C}, {D}, and {E} are all 1-itemsets, {AC} is a 2-itemset, and so on. All Frequent Itemset with min support =50% is ABDE, BCE are maximal-by-inclusion Frequent Itemsets i.e., they are not a subset of any other Frequent Itemset. The Table III contains Closed Itemsets and their minimal generators. Table IV lists 19 Association Rules extracted from the above Table I sample dataset.

TABLE II: Generated Frequent Itemsets

	Itemsets	Support
1	B	100%
2	E, BE	83%
3	A, C, D, AB, AE, BC, BD, ABE	66%
4	AD, CE, DE, ABD, ADE, BDE, BCE, ABDE	50%

TABLE III: Closed Itemsets And Minimal Generators

	Closed Itemsets	Minimal Generators	Support
1	B	B	1.00
2	BE	E	0.83
3	BD	D	0.66
4	BC	C	0.66
5	ABE	AB, AE	0.66

TABLE IV: Association Rules

	Association Rule	Support	Confidence
1	B→E	0.83	1.00
2	A→B, A→E, A→BE, C→B, D→B, E→B, AB→E, AE→B	0.66	1.00
3	AD→BE, DE→AD, ABD→E, ADE→B, BDE→A, CE→B, , DE→A, DE→B, AD→B, AD→E	0.50	1.00

From example of Table I dataset, the 19 Association Rules extracted, 19 are considered to be exact rules. Considering the MinMax exact basis and MinMax approximate basis, only 5 exact rules and 14 approximate rules, as shown in Tables V and VI, are derived and considered to be Non-Redundant.

TABLE V: Non-Redundant Rules Extracted Using Minmax Exact

	Association Rule	Support	Confidence
1	B→E	0.83	1.00
2	C→B, D→B, E→B, AB→E, AE→B	0.66	1.00

TABLE VI: Non-Redundant Rules Extracted Using Minmax Approximate

	Association Rule	Support	Confidence
1	A→B, A→E, A→BE	0.66	0.90
2	AD→BE, DE→AD, ABD→E, ADE→B, BDE→A, CE→B, , DE→A, DE→B, AD→B, AD→E	0.50	0.90

The Datasets used in these experiments were obtained from UCI KDD Machine Learning Repository (<http://kdd.ics.uci.edu/>). The Census - income dataset contains 32,561 transactions each of which describes the characteristics of Census-Income object. Each object has 15 attributes. They produce large numbers of Frequent Itemsets and thus a huge number of Association Rules even for very high values of support. Redundancy elimination is particularly important to these dense datasets. In this dataset we extracted 21 Non-Redundant rules out of 32,561 transactions with a confidence > 0.70. From the 21 Association rules given in the Table V, 13 rules are positive, which indicates that these rules the antecedent support the consequent, where as in the remaining 8 rules the antecedent is against the consequent. The no. of exact rules and redundancy elimination is given in Table VII.

TABLE VII Non-Redundant Rules Extracted Using Minmax Exact

S. No	Association Rules	Sup	Conf	CF
1	Race=White and Capital_Gain=0 and Capital_Loss=0 →Native_Community = US	0.74	0.92	0.03
2	Class=<=50K and Capital_Loss=0 →Native_Community = US	0.74	0.89	-0.06
3	Class=<=50K and Capital_Gain=0 →Native_Community = US	0.73	0.89	-0.06
4	Class=<=50K and Capital_Gain =0 and Capital_Loss = 0 →Native_Community = US	0.70	0.89	-0.07
5	Native_Community = US and Capital_Gain = 0 and Capital_Loss = 0 →Race = White	0.78	0.87	0.02
6	Class = <=50K and Capital_Loss = 0 → Race = White	0.74	0.84	-0.12
7	Class = <=50K and Capital_Gain = 0 → Race = White	0.73	0.84	-0.13
8	Class = <=50K and Capital_Gain = 0 and Capital_Loss = 0 → Race = White	0.70	0.84	-0.13
9	Native_Community = US and Capital_Gain = 0 and Capital_Loss = 0 → Class = <=50K	0.78	0.81	0.06
10	Race = White and Capital_Gain = 0 and Capital_Loss = 0 → Class = <=50K	0.74	0.80	0.05
11	Native_Community = US and Capital_Gain = 0 → Class = <=50K	0.82	0.79	0.04
12	Race =White and Native_Community=US and Capital_Gain= 0 → Class = <=50K	0.72	0.77	0.02
13	Race=White and Native_Community = US and Capital_Loss = 0 → Class = <=50K	0.75	0.75	-0.03
14	Race = White and Native_Community = US → Class = <=50K	0.79	0.74	-0.09
15	Class = <=50K and Capital_Gain = 0 and Capital_Loss = 0 → WorkClass = Private	0.70	0.72	0.03
16	Class = <=50K and Capital_Gain = 0 → WorkClass = Private	0.73	0.72	0.03
17	Class = <=50K and Capital_Loss = 0 → WorkClass = Private	0.74	0.72	0.03
18	Class = <=50K → WorkClass = Private	0.76	0.72	0.03
19	Race = White and Capital_Gain = 0 and Capital_Loss = 0 → WorkClass = Private	0.74	0.71	0.01
20	Capital_Gain = 0 and Capital_Loss = 0 → WorkClass = Private	0.87	0.71	0.01
21	Race = White and Capital_Gain = 0 → WorkClass = Private	0.78	0.70	0.01

The above rules are obtained by taking the minsupp as 70% and minconf as 70%. The above are non redundant rules with more antecedents and consequents as Zaki[15] generated rules are general rules which have shorten antecedent and consequent rules. The zaki[15] proposed work contains only frequent itemsets and closed itemsets which are still large as compared with the above generated rules.

Here we also given the rules importance with the help of certainty factor(CF) value, in this most of the rules are positive, thus many of the rule antecedent supports the consequent. Some of the rules CF values are negative which indicates that the antecedent denies the consequent. This is type of measure is not indicated in any previous proposed work.

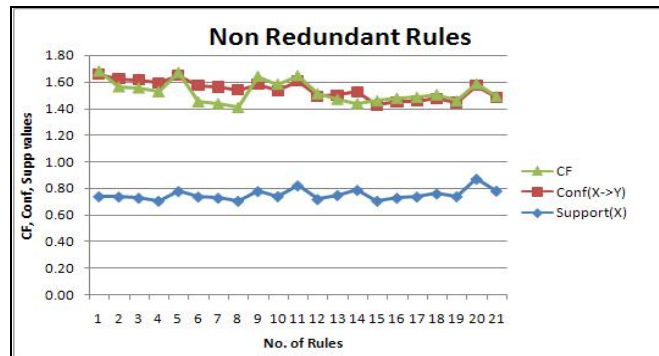


Fig 1. Supp, Conf, CF values for Non Redundant Rules for Census – Income Dataset

Conclusions

The challenging problem of generating the association rule mining is without redundancy rules in the extracted Association Rules. Which may affect the quality of the Association Rules and Frequent pattern generation in Non – Taxonomy Data sets.. With the above approach we can remove the redundant Frequent patterns using MinMax rules, to get quality Frequent patterns and then association rule mining.

REFERENCES

- [1] R. Agrawal and R. Srikant (1994), “Fast algorithms for mining Association Rules in large databases,” in *Proceedings of the 20th International Conference on Very Large Data Bases*, 1994, pp. 487–499.
- [2] N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal, (1999), “Discovering Frequent Closed Itemsets for Association Rules,” in *Database Theory - ICDT '99*, 7th International Conference, Jerusalem, Israel, January 10-12, 1999, Proceedings, 1999, pp. 398–416.
- [3] T. Calders and B. Goethals, (2002), “Mining all non-derivable Frequent Itemsets,” in *Proceedings of the 6th European Conference on Principles of Data Mining and Knowledge Discovery*, pp. 74–85.
- [4] X. Yan, H. Cheng, J. Han, and D. Xin, (2005), “Summarizing Itemset patterns: a profile based approach,” in *Proceedings of the Eleventh ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pp. 314–323.
- [5] C. Yang, U. M. Fayyad, and P. S. Bradley, (2001), “Efficient discovery of error-tolerant Frequent Itemsets in high dimensions,” in *Proceedings of the seventh ACM SIGKDD international conference on Knowledge discovery and data mining*, pp. 194–203.
- [6] D. Gunopulos, R. Khardon, H. Mannila, and H. Toivonen, (1997), “Data mining, hypergraph transversals, and machine learning,” in *Proceedings of the Sixteenth ACM SIGACTSIGMOD-SIGART Symposium on Principles of Database Systems*, 1997, pp. 209–216.
- [7] F. N. Afrati, A. Gionis, and H. Mannila, (2004), “Approximating a collection of Frequent sets,” in *Proceedings of the Tenth ACM SIGKDD International Conference on Know Discovery and Data Mining*, 2004, pp. 12–19.
- [8] J. Han, J. Wang, Y. Lu, and P. Tzvetkov, (2002), “Mining top-k Frequent closed patterns without minimum support,” in *Proceedings of the 2002 IEEE International Conference on Data Mining*, 2002, pp. 211–218.
- [9] D. Xin, H. Cheng, X. Yan, and J. Han, (2006), “Extracting redundancy-aware top-k patterns,” in *Proceedings of the Twelfth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 2006, pp. 444–453.
- [10] J. Pei, A. K. H. Tung, and J. Han, (2001), “Fault-tolerant Frequent pattern mining: Problems and challenges,” *ACM SIGMOD Workshop on Research Issues in Data Mining and Knowledge Discovery*, 2001.
- [11] H. Toivonen, M. Klememminen, P. Ronkaiven, K. Hatonen and H. Manika, (1995), “Pruning and grouping Discovered Association Rules”, In *ML Net wkshp on Statistics, Machine Learning and Discovering in Databases*, April 1995.
- [12] B. Ganter & R. Wille (1995), “Formal Concept analysis: Mathematical Foundations”, Springer Verlag.
- [13] N. Pasquier, Bastide, Y. Stemme G, & Lakhal, (2005), “Generating a Condensed Representation for Association Rule” *Journal of Intelligent system* 24(1), 29-60, 2005.
- [14] Calders T, Goethals B, (2002), “Mining all Non – Derivable Frequent Itemsets”, In *proceedings of the 6th European Conf. on principles of Data Mining and Knowledge discovery*, Vol 2431, pp 74 – 85, Springer – verlag 2002.
- [15] Zaki, M. J., (2004), “Mining Non – Redundant Association rules”, *Data Mining and Knowledge Discovery*, 9, 223–248, 2004
- [16] N. Pasquier, Bastide, Y. Touil R & Lakhal, “Efficient Mining of Association Rules using Closed Itemset”, *Information Sys.* (24), 25 – 46.
- [17] Shortliffe E. H. & Buchanan B. G., “A model of In exact Reasoning in Medicine”, *Mathematical Biosciences* 23 (3/4), 351 – 379.
- [18] Kryszkiewicz, Rybinski H, & Gajek M (2004), “Data less Transitions Between Concise Representation of Frequent Patterns”, *Journal of Intelligent Information system*, 22(1), 41 – 70.
- [19] Newell, Allen and Herbert A. Simon (March 1976). “Computer Science as Empirical Inquiry: Symbols and search” *Communications of the ACM* 19 (3). Retrieved 2006-12-21.
- [20] Pearl, J., “SCOUT: A Simple Game-Searching Algorithm With Proven Optimal Properties,” *Proceedings of the First Annual National Conference on Artificial Intelligence*, Stanford University, August 18-21, 1980, pp. 143-145.