IMPLEMENTATION OF FUZZY MULTI OBJECTIVE LINEAR PROGRAMMING FOR DECISION MAKING AND PLANNING UNDER UNCERTAINTY

C.KAVITHA
Research Scholar, Dept of Mathematics,
Sathyabama University, Chennai,
Tamil Nadu, India
ceekavi@gmail.com

C.VIJAYALAKSHMI
(SAS) Department of Mathematics
VIT University, Chennai
vijusesha2010@gmail.com

Abstract

This paper proposes a new hybrid algorithm for supplier selection and evaluation for quantitative, qualitative factors in supply chain management. Supplier selection and evaluation is formulated by integrating Multi-criteria decision making approach such as Analytic hierarchy process (AHP) improved by Rough set theory, Technique for order performance by similarity to ideal solution (TOPSIS) to rank the suppliers based on their overall performances and to allocate orders to the suppliers, to maximize the quality, service of the product, minimizing the cost weighted additive fuzzy multi objective linear program is used. Rough set theory is used to screen the suppliers by the decision makers. AHP was applied to determine the relative weights of evaluation criteria. TOPSIS was applied to rank suppliers in terms of their overall performances. Fuzzy approach is applied to overcome the vagueness, imprecision and uncertainty in real life situations .This method helps the decision maker to select the best supplier, to allocate orders to the suppliers with multiple products, multiple criteria in multi period time with budget, capacity and product demand in the market who meets all the requirements of supply chain management. To evaluate the algorithm a case study is done in high technology Company named Multi-Flex Lami-Print Ltd which manufactures Flexible Packaging materials in India. The result shows that the model is effective and applicable to industries.

***Keywords*: Rough set theory, Analytic Hierarchy Process, TOPSIS, Multi criteria decision making, Fuzzy multi objective linear programming, Supply chain Management.**

1. INTRODUCTION:

Supply Chain Management [SCM] consists of multiple suppliers, distributors, customers, manufacturers etc with functions such as purchasing, distribution and manufacturing etc in an organization. SCM is the integration of organization of information and logistic activities across firm for the purpose of delivering goods and services that provide value to the customers. Selection of suppliers and allocating orders to the suppliers is important to achieve the objectives in an organization. Supplier selection is a Multiple Criteria Decision Making Problem [MCDM] and it is important to calculate the tangible and intangible factors to find the best supplier. Supplier's performance will play a vital role in purchasing department because reduction of purchasing cost and maximizing the profit improves competitiveness among the firm. MCDM technique is adopted for selecting and ranking the suppliers subject to multiple criteria's or objectives. To overcome the vagueness and imprecision of goals, constraints and parameters in Fuzzy multi objective linear model an asymmetric fuzzy decision making technique is applied to enable the decision maker to assign different weights to various criteria (Amid et al., 2006). AHP improved by rough set theory and multi_objective mixed integer programming to determine the number of suppliers to employ the order quantity (Weijun Xia, Zhiming Wu., 2007). Lexicographic method is used to solve a piecewise linear quantity discount model for vendor selection problem (N.Arun kumar et al., 2006). Multi criteria decision making integrated using Taguchi loss functions, AHP and Fuzzy programming techniques to address the supplier selection problem in a closed-loop supply chain network (Satish Nukala and Surendra.M.Gupta., 2007). TOPSIS is used to find the tradeoff between tangible and intangible factors to calculate the rating of suppliers then by applying these ratings as coefficients in the fuzzy multi objective model and finally solved by single objective function (O.Jadidi et al., 2008). TOPSIS group decision making has been integrated with Fuzzy LP model to solve the supplier's selection and allocating the order quantities to the supplier (Jafar Razmi et al 2009). A Fuzzy bi_objective model is proposed for single item single [period supplier selection and purchasing problem under capacity constraint, supply uncertainty and budget limitation (Elhan Maghool and Jafar Razmi., 2010). Multi objective linear program utilized a Fuzzy Compromise program to convert the problem to a single objective model and incorporate the weights of objectives through various decision makers' opinion (Bein Elahi et al 2011). ANP and TOPSIS are used to calculate the weight, ranking the suppliers and then LP model is used to allocate the order quantity to each vendor (Chin-Tsai Lin et al., 2011). Two stage logarithmic goal programming method for generating weights from interval comparison matrices was used for ranking and selecting the suppliers then a fuzzy multi objective model was presented for quota allocation to suppliers (Mehdi Seifbarghy et al., 2011). Selection of suppliers is done by integrating AHP-TOPSIS method were decision criteria's were identified by Delphi method and order allocation to every selected suppliers done by multi objective linear programming model (Kambiz Shahroudi et al., 2011). Fuzzy multi objective linear program with fuzzy resources are used to allocate orders to the suppliers and selection of suppliers calculated by AHP improved by Rough set theory in multi period time horizon (H.Haleh, and A.Hamidi., 2011). In this paper we propose a new algorithm for a single objective weighted additive fuzzy linear program to solve the multi objective, multi products, multi period in order to calculate the optimum order quantities to each supplier where the weights of the objectives corresponds to the suppliers based on the previous experience are obtained by AHP improved with rough set theory, constraints includes both deterministic and fuzzy whereas fuzzy constraints are calculated by ranking method. The paper is organized as follows. Section 2 contains the basic information. Formulation of proposed algorithm is presented in section 3. Mathematical model and experimental data is presented in section 4. Conclusion is provided in section 5 and section 6 contains the references.

2. PRELIMINARIES:

2.1 ROUGH SET THEORY:

Rough set theory is a mathematical tool to treat the vague and imprecise. Here uncertain and imprecision is expressed by a boundary region of a set [Z.Pawlak.1991]. Rough set is defined by topological operations called approximations. Rough set theory addresses granularity of knowledge expressed by indiscernibility relation. It is intended to express the fact that not able to deal with single objects but we have to consider clusters of indiscernible objects.

2.2 INFORMATION SYSTEMS AND APPROXIMATIONS:

Consider the 4-tuple S = {U, R, V, f}, where U is a finite set of objects (universe); R = C $\cup$ D is a set of attributes, subsets C and D are the condition attribute set and the decision attribute set respectively, V_r is the domain of the attribute r, $V = \bigcup_{r \in R} V_r$ and $f : U \times R \rightarrow V$ is a total function such that $f(x, r) \in V_r$ for each r$\in$ R, x$\in$ U called information function.

To every non-empty subset B of attributes R (B $\subseteq$ R) is associated an indiscernibility relation on U denoted by IND (B). IND (B) $= \{(x, y)/(x, y) \in U \times U, \forall b \in B(b(x) = b(y))\}$ .It is defined as an equivalence relation (reflexive, symmetric and transitive). The family of all the equivalence classes of the relation IND (B) is denoted by U/IND (B).

2.3 ENTROPY:

Entropy H(P) of knowledge P (attributes set) is defined as H(P) = $-\sum_{i=1}^{n} p(X_i)\log_2 p(X_i)$ where $p(X_i)$ = $|X_i|/|U|$ and $p(X_i)$ denotes the probability of X_i when P is on the partition X = {X_1, X_2, …….X_n} of universe U, i = 1,2…………n.

2.4 CONDITIONAL ENTROPY:

Conditional entropy H(Q/P) which knowledge Q(U/IND(Q)={Y_1,Y_2,………..Y_m}) is relative to knowledge P(U/IND(P)={X_1,X_2,…………….X_n}) is defined as $H(Q/P) = -\sum_{i=1}^{n} p(X_i)\sum_{j=1}^{m} p(Y_j / X_i)\log_2 p(Y_j / X_i)$, where $p(Y_j / X_i)$ is conditional probability i = 1,2…n, j = 1,2……….m.

2.5 SIGNIFICANCE:

Suppose that decision table S = {U, R, V, f}, R = C $\cup$ D , subsets C and D are the condition attribute set and the decision attribute set respectively attribute subset A $\subset$ C. The attribute significance SGF (a, A, D) of attribute (a $\in$ C/A) is defined as $SGF(a, A, D) = H(D/A) - H(D/A \cup \{a\})$. Given attribute subset A, the greater the value of SGF (a, A, D), the more important attribute a is for decision D.

2.6 ANALYTIC HIERARCHY PROCESS [AHP]:

AHP is a multi_criteria decision method which reduces the complexity of decision making by constructing a hierarchical structure with decision elements. In order to determine the relative preferences for two elements a pair wise comparison matrix is constructed by the decision maker using standard preference scale in AHP is 1-9 scale which lies between ‘equal importance’ to ‘extreme importance’. To calculate the relative weights of elements in pair wise comparison matrix eigen value method is used. Consistency index is calculated to ensure that the judgments of decision makers are consistent. If CR is less than 0.1 then the judgments are consistent.

2.7 TECHNIQUE FOR ORDER PREFERENCE BY SIMILARITY TO IDEAL SOLUTION [TOPSIS]:

TOPSIS was founded by Hwang and Yoon. TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is a Multi attribute Decision Making [MADM] technique which was applied to manage real world problems which simultaneously considers the ideal and the anti-ideal solution for the corresponding attributes [K. Yoon, C.L. Hwang]. TOPSIS method is based on the concept that the best alternative among several alternatives that should have the shortest Euclidean distance from the positive ideal solution (PIS), and the farthest from the negative ideal solution (NIS).

2.8 MEMBERSHIP FUNCTION:

If the elements a_i(i=1,2…n) of a set $\tilde{A}$ are subset of universal set X then set $\tilde{A}$ can be represented for all elements xεX by its characteristic function $\mu_{\tilde{A}}(x) = \begin{cases} 1 \text{ if } x \in X \\ 0 \text{ otherwise} \end{cases}$

The function $\mu_{\tilde{A}}(x)$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each xεX is called a fuzzy set.

2.9 DEFINITION:

A fuzzy number $\tilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by $\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ \frac{x-c}{b-c}, b \le x \le c \\ 0 \text{ otherwise} \end{cases}$

2.10 DEFINITION:

A ranking function is a function $R : F(R) \to R$ where F(R) is a set of fuzzy numbers defined on set of real numbers which maps each fuzzy number into the real line where a natural order exists. Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number then $R(\tilde{A}) = \frac{a + 2b + c}{4}$.

2.11 MULTI OBJECTIVE LINEAR MODEL:

A general linear multi objective model can be presented as: Find a vector x written in the transformed form $x^T = [x_1, x_2, \ldots\ldots x_n]$ which minimizes objective function Z_k and maximizes objective function Z_l with

$$Z_k = \sum_{i=1}^{n} C_{ki} X_i, \ k = 1,2,\ldots\ldots\ldots\ldots p \ \text{and} \ Z_l = \sum_{i=1}^{n} C_{li} X_i, \ l = p+1, p+2 \ldots\ldots\ldots q$$

Subject to $x \in X_d$, $X_d = \left\{ x / g(x) = \sum_{i=1}^{n} a_{ri} x_i \le b_r, \ r = 1,2.....m, \ x \ge 0 \right\}$ where C_{ki}, C_{li}, a_{ri} and b_r are crisp or fuzzy values.

3. FUZZY MULTI OBJECTIVE LINEAR PROGRAMMING:

Allocating orders to suppliers involves many criteria's which are expressed in imprecise terms like 'Reasonable in prices', 'Excellent in quality', 'Good in service' etc. Such vagueness and imprecision cannot be determined by deterministic model and it is solved by fuzzy. Fuzzy Mathematical Program has the capable to handle Multi-objective problems, Decision making process, Vagueness of the linguistic type, uncertainty etc in fuzzy environment using membership functions. Membership functions of an objective function and constraints determines how close to its optimal value. Since the objectives have different priorities among the decision makers Weighted Fuzzy Multi Objective Programming is used.

Step 1: Supply chain hierarchy is formulated by defining the criteria, sub criteria and alternatives.

Step 2: To reduce the evaluation bias in decision table, the concept of rough set theory is used for information

regarding the objects supplied in the form of data table. Here the data set is represented as a table where each row represents an object and each column represents an attribute that can be measured for each object which is supplied by a human expertise. Thus decision for criteria and sub criteria's with respect to their important in reaching the goal were calculated by Rough set.

Step 3: To obtain the overall weights of each alternative for qualitative and quantitative factors of each supplier

in multiple criteria decision making problem, AHP is used by independently weight each alternative with respects to each of the decision Criteria. Combine Criteria Priorities with Alternative Priorities to determine an overall Priority of Alternatives.

Step 4: Suppliers ranking is done by TOPSIS method by integrating the supplier weights obtained by AHP

improved with Rough set theory for quantitative factors . In TOPSIS normalized decision matrix is calculated as .

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

Then the weighted normalized matrix is calculated as $V = w_i * r_{ij}$. Determine the ideal and negative ideal solution as

$$A^+ = \{(\max_i v_{ij} \mid j \in J), (\min_i v_{ij} \mid j \in J') \mid i = 1,2,...m\}$$

$$A^- = \{(\min_i v_{ij} \mid j \in J), (\max_i v_{ij} \mid j \in J') \mid i = 1,2,...m\}$$

Calculate the separation measure of positive ideal as $S_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}$ and negative ideal as $S_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}$. Calculate the relative closeness ideal solution as $C_i^* = \frac{S_i^-}{(S_i^+ + S_i^-)}$

Step 5: Construct multi objective model for allocating orders to the suppliers.

Step 6: Here objectives are fuzzy and constraints are fuzzy and deterministic in nature. In Constraints fuzzy

resources are triangular in number. To make it as a crisp model fuzzy ranking is done using the formula $R(\tilde{A}) = \frac{a + 2b + c}{4}$.

Step 7: Solve the MOLPP as a single objective linear program as in Zimmermann (1978), each time using one objective by ignoring others and determine the optimal solution. Each objective function find lower and upper bound corresponding to the set of solutions. For every objective function its value changes linearly from Z_j^- to Z_j^+ it may be considered as a fuzzy number with linear membership function $\mu_{z_j}(x)$.

Step 8: Formulate the equivalent crisp model.

Step 9: Since the fuzzy goals and constraints are of unequal importance, weighted additive model

is constructed. Weights of the objectives are calculated by integrating Rough set with Analytic hierarchy process.

4. Mathematical Model for Supplier Selection:

4.1 Notations:

i- index for suppliers, i= 1,2……m
j- index for periods, j= 1,2………n
k- index for products, k= 1,2……..p
x_{ijk} - Order quantity of kth product from ith supplier in jth period.
q_{ijk} -quality level of kth product purchased from ith supplier in jth period.
t_{ijk} - on-time delivery rate of kth product purchased from ith supplier in jth period.
D_k - fuzzy demand quantity of kth product.
P_{ijk} – Unit price of the kth product from the ith supplier in jth period.
B_{kj} –purchasing budget of the kth product in jth period.
MC_{ijk} –Maximum supply capacity of the kth product from the ith supplier in jth period.
Q_{kmax} –buyer's maximum acceptable defective rate of kth product.
T_{kmin} –buyer's minimum acceptable on time delivery rate on kth product.

4.2 Model formulation:

To allocate the optimum order quantities to the suppliers, we use a MOLP model (C. Kavitha and C. Vijayalakshmi., 2012) with three objectives to optimize total purchasing costs, quality and and service. Consider purchasing budget, production demand, suppliers' capacity, and quality control and delivery reliability control constraints. The objective functions are as follows:

Purchasing cost: To minimize the total purchasing cost. Purchasing cost includes the price, transportation cost and ordering cost.

$$Min\ Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk} \leq \sim Z_1^0$$

Quality: To maximize the number of non defective items for improving product quality.

$$Max\ Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} q_{ijk}x_{ijk} \leq \sim Z_2^0$$

Service: To maximize the number of items delivered on time.

$$Max\ Z_3 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} t_{ijk}x_{ijk} \leq \sim Z_3^0$$

Constraints are as follows:

Purchasing Budget Constraints: Total purchasing payment for each product cannot exceed the budget of each product in that period.

$$\sum_{i=1}^{m}\sum_{k=1}^{p} p_{ijk}x_{ijk} \leq B_{kj},\ j = 1,2...n$$

Demand Constraints: The assigned order quantity of each product from all suppliers must meet the demand quantity of each product in the total period.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk} \geq \tilde{D}_k$$

Capacity Constraints: The order quantity of the kth product from the ith supplier cannot exceed each supplier's capacity.

$x_{ijk} \leq \mathrm{MC}_{\mathrm{ijk}}$, i = 1,2.........m, j = 1,2,...........n, k = 1,2............p

Quality Control Constraints: The total defect quantity of each product cannot exceed maximum acceptable defective quantity of each product

$$\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ijk}\ \mathrm{x}_{\mathrm{ijk}} \leq \mathrm{Q}_{\mathrm{kmax}}\tilde{D}_k,\ \mathrm{k} = 1,2..............\mathrm{p}$$

Delivery Constraint: The total late delivery on each product cannot exceed minimum acceptable late delivery.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ijk}\ \mathrm{x}_{\mathrm{ijk}} \leq T_{k\min}\tilde{D}_k,\ \ \mathrm{k} = 1,2.......\mathrm{p}$$

Variable non-negativity Constraints: Non-negativity restrictions on the decision variables is

$x_{ijk} \geq 0$, i = 1,2...........m, j = 1,2..........n, k = 1,2.............p .

In constraints fuzzy resources are in triangular numbers they are converted to crisp value using the formula $R\left(\tilde{A}\right) = \frac{a+2b+c}{4}$. Then the above multi objective linear supplier selection model is solved as a single objective linear model using each time one objective. The values obtained for maximum and minimum objectives are kept as upper bound and lower bound in order to fuzzify the objectives and constraint. Membership functions are calculated for fuzzy goals and fuzzy constraint. Here fuzzy goals and fuzzy constraint have unequal importance to DM therefore weighted additive model is used in vector _objective optimization problem. Weights are multiplied to each membership function of fuzzy goals and then adding the results together to obtain a linear weighted utility function. Equivalent crisp model is

Max $w_1\lambda_1 + w_2\lambda_2 + w_3\lambda_3$

Subject to

$$\lambda_1 \leq [\ \mathrm{Z}_{\mathrm{k}}^{+} - (\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk})\ ]\ /\ \left[Z_k^{+}\ \text{-}\ \mathrm{Z}_{\mathrm{k}}^{\text{-}}\right]$$

$$\lambda_2 \leq [\ \mathrm{Z}_{\mathrm{l}}^{+} - (\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} q_{ijk}x_{ijk})\ ]\ /\ \left[Z_l^{+}\ \text{-}\ \mathrm{Z}_{\mathrm{l}}^{\text{-}}\right]$$

$$\lambda_3 \leq [\ \mathrm{Z}_{\mathrm{m}}^{+} - (\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} t_{ijk}x_{ijk})\ ]\ /\ \left[Z_m^{+}\ \text{-}\ \mathrm{Z}_{\mathrm{m}}^{\text{-}}\right]$$

$$x_{ijk} \leq \mathrm{MC}_{\mathrm{ijk}}\ ,$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ijk}\ \mathrm{x}_{\mathrm{ijk}} \leq \mathrm{Q}_{\mathrm{kmax}}\tilde{D}_k\ ,$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ijk}\ \mathrm{x}_{\mathrm{ijk}} \leq T_{k\min}\tilde{D}_k\ ,$$

$$x_{ijk} \geq 0\ .$$

$$\lambda_i \in (0,1)$$

5. APPLICATION

Supply chain management has applied to a professionally managed company namely Multi-Flex Lami-Print Ltd., who manufactures Quality Flexible Packaging Materials against specific orders from their customers like Hindustan Unilever Ltd., ITC Ltd., Tata Tea Ltd., Cavinkare Pvt Ltd….. will produce important raw materials like Polyester film, Bi-axially oriented Poly Propylene film, Polyethylene film and also Printing inks, Lamination adhesives, Diluting Solvents……….from the best suitable supplier's for various production processes such as Printing-Lamination-Slitting-Finishing.

Before selecting / finalizing a best suitable, ideal, reliable supplier from many of them in that category, Multi-Flex will carefully analyze important factors/ parameters in many of their suppliers suiting to their requirements. Finally suppliers meeting / fulfilling all their requirements in terms of most important basic criteria like Cost-Quality-Service will be selected from many of the suppliers in that category.

Basic process of manufacturing Polyester film is Polyester chips will be coextruded to bi-axially oriented thin film in the range of 10m to 200m for the various applications. From these range only 10micron and 12micron thickness of polyester film only are used in packaging industry.

Suppliers of Polyester film are Garware Polyesters Ltd (Supplier 1), Polyplex Ltd (Supplier 2), Jindal Films Ltd (Supplier 3), and U Flex Films Ltd (Supplier 4).

Price: Currently the basic price of the film is almost same in all Indian suppliers market. Better volume discount about 10% is offered for more than 25 MT off-take in U Flex Ltd(Supplier 4)., Also U Flex Ltd is situated in Tax exempted area ie Noida, UP, hence 2% CST will be lesser in U Flex. Since U Flex's production capacity is huge, their conversion and contribution(profit margin) will be slightly lower than other suppliers, that price advantage will be passed on to the Customers like us. Also U Flex offers Quick payment discount ie if we make the payment before the agreed credit days, for the differential days, discount is offered in pro-rata basis.

Quality: Highly sophisticated precision machineries are required for maintain uniform thickness of the film throughout the production lot/batch. Sincere dedicated Quality control, Quality assurance team and the testing equipments are required for checking the quality aspects/parameters of their inward raw material and outward finished material ie Polyester film. Optical property of Polyester film is very important which will ensure clarity of film. Surface treatment level of the film should be optimal which will facilitate proper perfect printing. Heat resistant property of film to be adequate by using/maintaining proper recipe of the raw material. QC/QA & Analysis Certificate to be given for all the supplies to the Customers like us to ensure specifications is met as per our requirement.

Services: Supplier should ensure, any of the quality complaint from the Customer should be attended/addressed immediately. Defect evaluation to be done and the necessary Corrective action to be taken immediately to ensure Defect free supplies. U Flex's quality management is in order and they are trying to achieve Zero defect supplies. Some suppliers like Garware, U Flex will obtain feedback on quality of the materials from the Customers which will be helpful for the supplier to improve the quality still better. U Flex executives will get in touch with Multi-Flex regularly for getting feedback/comments if any on quality and services.

5.1 Selection Criteria for Suppliers

The level of hierarchy which we have taken here is four levels which include objectives, different decision criteria, attributes and the decision alternatives. The main objective here is the selection of best supplier for a manufacturing firm. Criteria which are considered here is Cost of the product, Quality of the product and Service performance of the supplier. Criteria's are decomposed into various attributes. Attributes considered are Product cost, Period cost, Standardization, Research and development, Defects, Preventive action, on time delivery, Response speed, Flexibility and after sales service. Criteria's are denoted by C_i, attributes are denoted by A_j and alternatives are denoted by S_k where (i, j, k = 1,2,…….). The overall objective is placed at level 1, criteria at level 2, attributes at level 3 and decision alternatives at level 4 can be seen in Figure1.

5.2 Cost of the product (C_1)

One of the important criteria in supply chain management is assessing the suppliers cost. Maximization of profit can be achieved by minimizing the cost. Factors affecting these criteria can be stated as follows:

5.2.1 Product cost (A_1):

To increase the profitability firm always requires the minimum price for the product. It includes each of the manufacturing cost elements [direct materials, direct labor and manufacturing overhead], processing cost, warranty cost and other cost which determines the total cost of the product. These are recorded as inventory and not expensed to cost of goods sold until the time of sale.

5.2.2 Period cost (A_2):

These are identifiable with a specific time period and these are non manufacturing cost which includes selling and administrative expenses.

5.3 Quality of the Product (C_2)

Most important factor is the high quality products which must be able to fulfill the customer's need and satisfaction. Quality of the product is measured by following attributes:

5.3.1 Standardization (A_3):

Standardized products are valuable and it is safe, secure, high quality, flexible etc which reinforces positive consumer perceptions of the product.

5.3.2 Research and Development (A_4):

It is an activity of new product development, to discover, to exploit the power of knowledge and to know about scientific and technological topics for the purpose of uncovering and enabling development of valuable new products, processes, and services. To satisfy customer and get their appreciation Research and development plays a vital role.

5.3.3 Defects (A_5):

It is very important that the products must be free of defect which improves the demand of the product in the market.

5.4 Service performance of supplier (C_3)

Performance of the supplier for a particular product in the market will increase the customer level which will be analyzed based on the following factors:

5.4.1 Preventive action (A_6):

Maximizing the profit of an industry is a long term risk cost; action plans must be taken to reduce the nonconformities and to improve the industry.

5.4.2 On time delivery (A_7):

To avoid the delay and lead time, suppliers must follow the predefined delivery schedule. If the customers receive the ordered things at time they will have an overall satisfaction to the suppliers.

5.4.3 Response speed (A_8):

Suppliers who will response to change based on customers demand, ordering the products, cost and quality of the product etc will have a good relation with customers. It will improve the performance of the supplier to the management.

5.4.4 Flexibility (A_9):

Suppliers must have the ability to provide flexible services to the customers and to manufacture the process as demanded by the customers.

5.4.5 After sales service (A_{10}):

Suppliers must have a long term relationship with customers which will increase the overall performance in the supply chain management.

Criteria’s such as price, quality, and service are evaluated for suppliers using attribute significance in rough set theory and they are rated using three-point scale such as the values of 1, 2 and 3 for “low”, “middle” and “high” for price criterion, quality and service criteria are associated with “good”, “middle” and “poor”. Then we list different combinations of criteria rates and it is given to evaluation team to make decision. In decision column number 1 represents “supplier is selected” and number 0 represents “supplier is not selected”. Table 1 represents the decision table for price, quality and service.

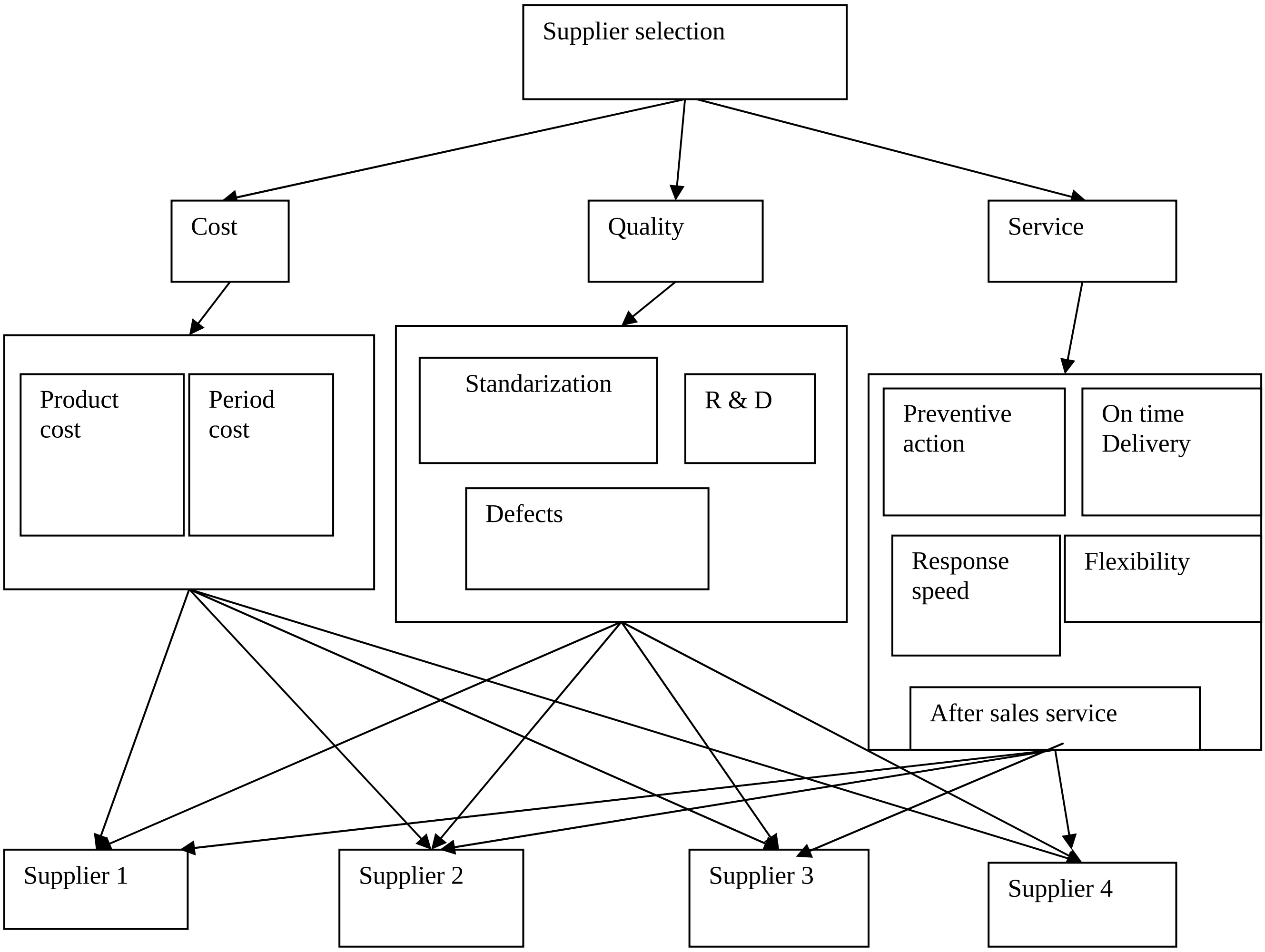


Figure 1: Hierarchical structure of Supplier selection

Table 1: Decision table about price, quality and service

U	Price(a)	Quality(b)	Service(c)	Decision(d)
1	3	2	1	0
2	2	3	1	0
3	1	2	3	0
4	3	1	2	0
5	1	3	2	0
6	2	1	3	0
7	1	1	3	1
8	3	1	1	0
9	1	3	1	0
10	2	2	3	0
11	3	2	2	0
12	1	1	1	1
13	2	2	2	1
14	3	2	3	0
15	3	3	2	0
16	1	2	2	1
17	2	1	2	1
18	2	2	1	1
19	1	1	2	1
20	1	2	1	1
21	2	1	1	1
22	1	3	3	0

Significances of price, quality and service by the following process:

U / IND {a, b, c} = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}, {15}, {16}, {17}, {18}, {19}, {20}, {21}, {22}},

U / IND {d} = {{1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 22}, {7, 12, 13, 16, 17, 18, 19, 20, 21}}

= $\{Y_1, Y_2\}$,

To find Price (a):

U / IND {b, c} = {{8, 12, 21}, {11, 13, 16}, {22}, {1, 18, 20}, {4, 17, 19}, {6, 7}, {2, 9}, {5, 15}, {3, 10, 14}}

$P(X_1) = 3 / 22$ $\quad P(Y_1 / X_1) = 1 / 3$ $\quad P(Y_2 / X_1) = 2 / 3$
$P(X_2) = 3 / 22$ $\quad P(Y_1 / X_2) = 1 / 3$ $\quad P(Y_2 / X_2) = 2 / 3$
$P(X_3) = 1 / 22$ $\quad P(Y_1 / X_3) = 1 / 1$ $\quad P(Y_2 / X_3) = 0 / 1$
$P(X_4) = 3 / 22$ $\quad P(Y_1 / X_4) = 1 / 3$ $\quad P(Y_2 / X_4) = 2 / 3$
$P(X_5) = 3 / 22$ $\quad P(Y_1 / X_5) = 1 / 3$ $\quad P(Y_2 / X_5) = 2 / 3$
$P(X_6) = 2 / 22$ $\quad P(Y_1 / X_6) = 1 / 2$ $\quad P(Y_2 / X_6) = 1 / 2$
$P(X_7) = 2 / 22$ $\quad P(Y_1 / X_7) = 2 / 2$ $\quad P(Y_2 / X_7) = 0 / 2$
$P(X_8) = 2 / 22$ $\quad P(Y_1 / X_8) = 2 / 2$ $\quad P(Y_2 / X_8) = 0 / 2$
$P(X_9) = 3 / 22$ $\quad P(Y_1 / X_9) = 3 / 3$ $\quad P(Y_2 / X_9) = 0 / 3$

SGF (a, {b, c}, {d}} = H ({d}/ {b, c}) – H ({d}/ {a, b, c})

$$= -\frac{3}{22}\left\{\frac{1}{3}\log_2\left(\frac{1}{3}\right)+\frac{2}{3}\log_2\left(\frac{2}{3}\right)\right\}\times 4-\frac{3}{22}\left(\frac{1}{2}\log_2\left(\frac{1}{2}\right)+\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right)$$

= 0.19172

To find Quality (b):

U / IND {a, c} = {{9, 12, 20}, {13, 17}, {14}, {2, 18, 21}, {5, 16, 19}, {3, 7, 22}, {1, 8}, {4, 11, 15}, {6, 10}}

$P(X_1) = 3 / 22$ $P(Y_1 / X_1) = 1 / 3$ $P(Y_2 / X_1) = 2 / 3$
$P(X_2) = 2 / 22$ $P(Y_1 / X_2) = 0 / 2$ $P(Y_2 / X_2) = 2 / 2$
$P(X_3) = 1 / 22$ $P(Y_1 / X_3) = 1 / 1$ $P(Y_2 / X_3) = 0 / 1$
$P(X_4) = 3 / 22$ $P(Y_1 / X_4) = 1 / 3$ $P(Y_2 / X_4) = 2 / 3$
$P(X_5) = 3 / 22$ $P(Y_1 / X_5) = 1 / 3$ $P(Y_2 / X_5) = 2 / 3$
$P(X_6) = 3 / 22$ $P(Y_1 / X_6) = 2 / 3$ $P(Y_2 / X_6) = 1 / 3$
$P(X_7) = 2 / 22$ $P(Y_1 / X_7) = 2 / 2$ $P(Y_2 / X_7) = 0 / 2$
$P(X_8) = 3 / 22$ $P(Y_1 / X_8) = 3 / 3$ $P(Y_2 / X_8) = 0 / 3$
$P(X_9) = 2 / 22$ $P(Y_1 / X_9) = 2 / 2$ $P(Y_2 / X_9) = 0 / 2$

SGF (b, {a, c}, {d}} = H ({d}/ {a, c}) – H ({d}/ {a, b, c})

$$= -\frac{3}{22}\left\{\frac{1}{3}\log_2\left(\frac{1}{3}\right)+\frac{2}{3}\log_2\left(\frac{2}{3}\right)\right\}\times 3-\frac{3}{22}\left(\frac{2}{3}\log_2\left(\frac{2}{3}\right)+\frac{1}{3}\log_2\left(\frac{1}{3}\right)\right)$$

= 0.15068

To find Service (c):

U / IND {a, b} = {{7, 12, 19}, {10, 13, 18}, {15}, {6, 17, 21}, {3, 16, 20}, {5, 9, 22}, {4, 8}, {1, 11, 14}, {2}}

$P(X_1) = 3 / 22$ $P(Y_1 / X_1) = 0 / 3$ $P(Y_2 / X_1) = 3 / 3$
$P(X_2) = 3 / 22$ $P(Y_1 / X_2) = 1 / 3$ $P(Y_2 / X_2) = 2 / 3$
$P(X_3) = 1 / 22$ $P(Y_1 / X_3) = 1 / 1$ $P(Y_2 / X_3) = 0 / 1$
$P(X_4) = 3 / 22$ $P(Y_1 / X_4) = 1 / 3$ $P(Y_2 / X_4) = 2 / 3$
$P(X_5) = 3 / 22$ $P(Y_1 / X_5) = 1 / 3$ $P(Y_2 / X_5) = 2 / 3$
$P(X_6) = 3 / 22$ $P(Y_1 / X_6) = 3 / 3$ $P(Y_2 / X_6) = 0 / 3$
$P(X_7) = 2 / 22$ $P(Y_1 / X_7) = 2 / 2$ $P(Y_2 / X_7) = 0 / 2$
$P(X_8) = 3 / 22$ $P(Y_1 / X_8) = 3 / 3$ $P(Y_2 / X_8) = 0 / 3$
$P(X_9) = 1 / 22$ $P(Y_1 / X_9) = 1 / 1$ $P(Y_2 / X_9) = 0 / 1$

SGF (c, {a, b}, {d}} = H ({d}/ {a, c}) – H ({d}/ {a, b, c})

$$= -\frac{3}{22}\left\{\frac{1}{3}\log_2\left(\frac{1}{3}\right)+\frac{2}{3}\log_2\left(\frac{2}{3}\right)\right\}\times 3$$

= 0.11301

Pair wise comparison judgment matrix is constructed to criteria significance as

$$J = \begin{bmatrix} 1 & 1.2724 & 1.6965 \\ 0.7859 & 1 & 1.3333 \\ 0.5895 & 0.75 & 1 \end{bmatrix}$$

Judgment matrix is then translated into the eigen value problem and the normalized relative weight is $\begin{bmatrix} 0.4209 & 0.4209 & 0.4209 \\ 0.3308 & 0.3308 & 0.3308 \\ 0.2482 & 0.2482 & 0.2482 \end{bmatrix}$.

Then $\lambda_{\max} = 2.3754 * 0.4209 + 3.0224 * 0.3308 + 4.0298 * 0.2482 = 3$

Consistency index (CI) is $CI = \frac{\lambda_{\max} - n}{\text{n - 1}} = \frac{3 - 3}{2} = 0$. This shows that pair wise comparison matrix constructed by rough set method possesses complete consistency.

Table 3 denotes the decision table of preference to cost sub-criteria; Table 5 denotes the decision table of preference to quality sub-criteria; Table 7 denotes the decision table of preference to service sub-criteria. Table 2 shows the computational results according to the decision in Table1. Table 4 shows the computational results according to the decision in Table3. Table 6 shows the computational results according to the decision in Table 5. Table 8 shows the computational results according to the decision in Table 7.

Table 2: Computational results about price, quality and service

	SGF	Weight	Λ_{max}	CI
Price(a)	0.19172	0.4209	3.0000	0
Quality(b)	0.15068	0.3308		
Service (c)	0.11301	0.2482		

Sub Criteria's for cost are evaluated for suppliers using attribute significance in rough set theory and they are rated using three-point scale such as the values of 1, 2 and 3 for "high", "middle" and "low" for product cost and period cost are associated with values of 1,2 and 3 for "high", "middle" and "low". Then we list different 7 combinations of criteria rates and it is given to evaluation team to make decision.

Table 3: Decision table about price sub-criteria

U	Product cost(d)	Period cost(e)	Decision(d)
1	3	3	0
2	1	2	0
3	3	2	1
4	2	2	0
5	3	1	1
6	1	1	0
7	2	1	1

Table 4: Computational results about price sub-criteria

	SGF	Weight	Λ_{max}	CI
Product cost	0.2368	0.5368	2.0000	0
Period cost	0.2043	0.4632		

Sub Criteria's for quality are evaluated for suppliers using attribute significance in rough set theory and they are rated using three-point scale such as the values of 1, 2 and 3 for "high", "middle" and "low" for standardization, Research and Development are associated with values of 1,2 and 3 for "low", "middle" and "high", Defects are associated with values of 1,2 and 3 for "high", "middle" and "low". Then we list different 18 combinations of criteria rates and it is given to evaluation team to make decision.

Table 5: Decision table about quality sub-criteria

U	Standardization(f)	Research & Development(g)	Defects (h)	Decision(d)
1	2	3	3	1
2	1	2	1	0
3	1	3	3	1
4	1	1	2	0
5	1	1	1	0
6	1	2	2	0
7	1	3	2	1
8	1	2	3	1
9	1	2	2	0
10	1	1	3	0
11	2	3	2	1
12	2	2	2	0
13	2	3	1	0
14	2	2	1	0
15	2	2	3	0
16	2	1	1	0
17	2	1	2	0
18	2	1	3	0

Table 6: Computational results about quality sub-criteria

	SGF	Weight	Λ_{max}	CI
Standardization(f)	0.03344	0.1024	3.0000	0
Research & Development(g)	0.1926	0.5900		
Defects (h)	0.1004	0.3076		

Sub Criteria’s for service are evaluated for suppliers using attribute significance in rough set theory and they are rated using three-point scale such as the values of 1, 2 and 3 for “excellent”, “good” and “bad” for preventive action, on time delivery are associated with values of 3, 2 and 1 for “fast”, “moderate” and “slow”, response speed are associated with values of 1,2 and 3 for “high”, “average” and “good”, flexibility are associated with values of 3,2 and 1 for “good”, “average” and “bad”, after sales service are associated with values of 1,2 and 3 for “high”, “medium” and “low”. Then we list different 60 combinations of criteria rates and it is given to evaluation team to make decision.

Table 7: Decision table about service sub-criteria

U	Preventive action (i)	On time delivery(j)	Response speed(k)	Flexibility(l)	After sales service(m)	Decision(d)
1	1	3	1	3	1	1
2	2	3	1	3	1	1
3	1	3	2	3	1	1
4	1	2	1	3	1	0
5	2	3	1	1	2	0
6	1	3	1	1	2	0
7	1	3	1	1	1	0
8	2	3	1	1	1	0
9	1	3	1	2	2	1
10	1	3	2	1	2	0
11	1	3	1	2	1	1
12	1	3	2	1	1	0

13	1	3	1	3	2	1
14	1	3	2	2	2	0
15	1	2	1	1	2	0
16	1	2	1	1	1	0
17	1	2	1	2	2	0
18	1	3	2	2	1	1
19	1	2	2	1	2	0
20	1	2	2	1	1	0
21	1	3	2	3	2	0
22	1	2	1	2	1	1
23	1	2	2	2	2	0
24	1	2	1	3	2	0
25	2	3	2	1	2	0
26	2	3	2	1	1	0
27	1	1	2	1	2	0
28	1	1	2	1	1	0
29	1	2	2	3	1	1
30	1	2	2	2	1	0
31	1	2	2	3	2	0
32	1	1	1	1	2	0
33	1	1	1	1	1	0
34	2	2	1	3	1	1
35	2	3	2	2	2	0
36	2	3	1	2	2	0
37	1	1	2	2	2	0
38	1	1	1	2	2	0
39	2	2	2	3	1	0
40	2	3	2	2	1	0
41	2	3	1	2	1	0
42	1	1	2	2	1	0
43	1	1	1	2	1	0
44	2	1	1	3	1	0
45	2	1	2	3	1	0
46	2	3	2	3	2	0
47	2	3	1	3	2	1
48	1	1	2	3	2	0
49	1	1	1	3	2	0
50	2	3	2	3	1	1
51	2	2	1	1	2	0
52	2	2	1	3	2	0
53	2	2	1	2	1	0
54	1	1	2	3	1	0
55	2	2	1	2	2	0
56	1	1	1	3	1	0
57	2	2	1	1	1	0
58	2	2	2	3	2	0
59	2	1	1	3	2	0
60	2	1	2	3	2	0

Table 8: Computational results about service sub-criteria

	SGF	Weight	Λ_{max}	CI
Preventive action(i)	0.1004	0.1049	5.0000	0
On time delivery(j)	0.3114	0.3256		
Response speed (k)	0.1128	0.1179		
Flexibility (l)	0.3114	0.3256		
After sales service(m)	0.12042	0.1259		

Table 9 determined the global weights for all criteria and sub-criteria. Supplier’s quantitative information relative to criteria and sub-criteria are presented in Table 10.

Table 9: Composite priority weights for criteria and sub criteria

Criteria	Local weights	Sub criteria	Local weights	Global weights	Priority order
Price	0.4209	Product cost	0.5368	0.2259	1
		Period cost	0.4632	0.1949	3
Quality	0.3308	Standardization	0.1024	0.0339	7
		Research & Development	0.5899	0.1951	2
		Defects	0.3076	0.1018	4
Service	0.2482	Preventive action	0.1049	0.0260	10
		On time delivery	0.3256	0.0808	5
		Response speed	0.1179	0.0293	9
		Flexibility	0.3256	0.0808	6
		After sales service	0.1259	0.0312	8

Table 10: Supplier’s quantitative information

Suppliers	Purchasing cost		Standardization (%)	Research and Development (grade)	Defects (rate)	Preventive Action (rate)	On time delivery (rate)	Response Speed (rate)	Flexibility (%)	After sales service (grade)
	Product cost	Period cost								
1	32	33	85	2	0.03	0.75	0.15	0.7	90	3
2	22	23	90	1	0.04	0.85	0.09	0.8	95	4
3	25	25	95	2	0.03	0.95	0.17	0.9	80	4
4	27	27	75	3	0.04	0.90	0.15	0.95	70	3

Supplier’s ratings are calculated by forming pair wise comparison matrix and computing eigenvector on different criteria and sub criteria. Results are showed in Table 11. In order to calculate the final score of each supplier the weights of criteria, sub criteria and suppliers rating should be combined. It is performed by summing the product of supplier’s ratings and global weights. Thus the supplier’s weights are calculated by AHP.

Table 11: Supplier’s final weight

Suppliers	Purchasing cost		Standardization (%)	Research and Development (grade)	Defects (rate)	Preventive Action (rate)	On time delivery (rate)	Response Speed (rate)	Flexibility (%)	After sales service (grade)	Supplier weights
	Product cost	Period cost									
1	0.3019	0.3056	0.2464	0.25	0.2143	0.2174	0.2679	0.2089	0.2687	0.2143	0.2685
2	0.2075	0.2129	0.2609	0.125	0.2857	0.2464	0.1607	0.2388	0.2836	0.2857	0.2089
3	0.2358	0.2315	0.2753	0.25	0.2143	0.2754	0.3036	0.2687	0.2388	0.2857	0.2461
4	0.2547	0.2499	0.2174	0.375	0.2857	0.2609	0.2679	0.2836	0.2089	0.2143	0.2762

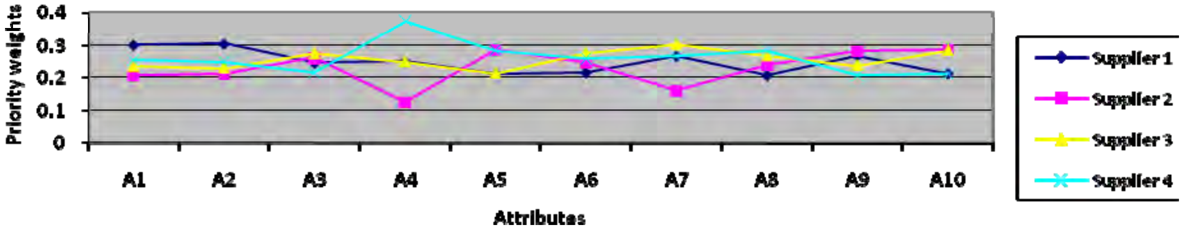


Figure 2: Weight of each supplier with respect to the attribute

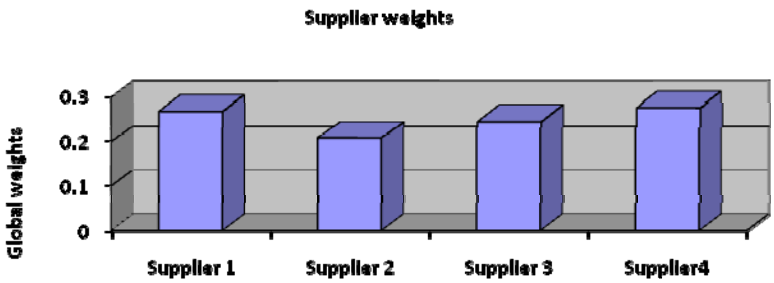


Figure 3: Supplier's Final weight with respect to qualitative and quantitative factors

TOPSIS method evaluates the quantitative values by constructing the normalized decision matrix. Decision matrix is normalized and it is shown in Table 12 and 13.

Table 12: Supplier's decision matrix using TOPSIS

Suppliers	Purchasing cost: Product cost	Purchasing cost: Period cost	Standardization (%)	Research and Development (grade)	Defects (rate)	Preventive Action (rate)	On time delivery (rate)	Response Speed (rate)	Flexibility (%)	After sales service (grade)
1	1024	1089	7225	4	0.0009	0.5625	0.0225	0.49	8100	9
2	484	529	8100	1	0.0016	0.7225	0.0081	0.64	9025	16
3	625	625	9025	4	0.0009	0.9025	0.0289	0.81	6400	16
4	729	729	5625	9	0.0016	0.81	0.0225	0.9025	4900	9
$\sum x_{ij}^2$	2862	2972	29975	18	0.005	2.9975	0.082	2.8425	28425	50
$\left(\sum x^2\right)^{\frac{1}{2}}$	53.49	54.52	173.13	4.24	0.07	1.73	0.29	1.69	168.59	7.07

Table 13: Supplier's normalized decision matrix using TOPSIS

Suppliers	Purchasing cost: Product cost	Purchasing cost: Period cost	Standardization (%)	Research and Development (grade)	Defects (rate)	Preventive Action (rate)	On time delivery (rate)	Response Speed (rate)	Flexibility (%)	After sales service (grade)
1	0.59	0.61	0.49	0.47	0.43	0.43	0.52	0.41	0.53	0.42
2	0.41	0.42	0.52	0.24	0.57	0.49	0.31	0.47	0.56	0.57
3	0.47	0.46	0.55	0.47	0.43	0.55	0.59	0.53	0.47	0.57
4	0.50	0.49	0.43	0.71	0.57	0.52	0.52	0.56	0.42	0.42

Suppliers weighted normalized decision matrix is constructed by weights obtained in Table 11 is used in conjunction with the previous normalized decision matrix to determine the weighted normalized matrix V . as in Table 14.

Table 14: Supplier's weighted normalized decision matrix using TOPSIS

Suppliers	Purchasing cost		Standardization (%)	Research and Development (grade)	Defects (rate)	Preventive Action (rate)	On time delivery (rate)	Response Speed (rate)	Flexibility (%)	After sales service (grade)
	Product cost	Period cost								
1	0.1584	0.1638	0.1316	0.1262	0.1155	0.1155	0.1396	0.1101	0.1423	0.1128
2	0.0856	0.0877	0.1086	0.0501	0.1191	0.1024	0.0648	0.0982	0.1169	0.1191
3	0.1157	0.1132	0.1354	0.1157	0.1058	0.1354	0.1452	0.1304	0.1157	0.1403
4	0.1381	0.1353	0.1188	0.1961	0.1574	0.1436	0.1436	0.1547	0.1160	0.1160

Then calculate the separation measure. Here n-dimensional Euclidean distance is used to measure the separation distances of each alternative to the ideal solution and negative-ideal solution. The values are in Table 15 and 16.

Table 15: Supplier's positive ideal solution using TOPSIS

Suppliers	$\sum(V_j^+ - V_{ij})^2$	$\left(\sum(V_j^+ - V_{ij})^2\right)^{\frac{1}{2}}$
1	0.0102	0.1009
2	0.0470	0.2168
3	0.0149	0.1221
4	0.0028	0.0529

Table 16: Supplier's negative ideal solution using TOPSIS

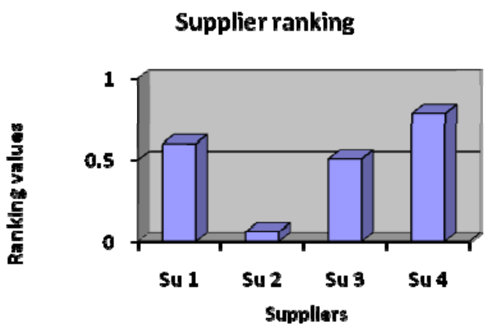
Suppliers	$\sum(V_j^- - V_{ij})^2$	$\left(\sum(V_j^- - V_{ij})^2\right)^{\frac{1}{2}}$
1	0.0241	0.1552
2	0.0002	0.0141
3	0.0159	0.1261
4	0.0402	0.2005

Then calculate the relative closeness to the ideal solution which is obtained in Table 17. Then finally rank the preference order which has the shortest distance to the ideal solution.

Table 17: Supplier's closeness ideal solution using TOPSIS

Suppliers	$C_i^* = \frac{S_i^-}{(S_i^+ + S_i^-)}$	Ranking
1	0.6060	2
2	0.0611	4
3	0.5081	3
4	0.7912	1

Figure 4: Supplier's ranking



Quantitative information related to Cost, Quality, Service, Capacity, Demand, Budget are said to be de-fuzzified data's corresponding to each criteria given in Table 18. The demand is a fuzzy number and is predicted to be about 495000,500000,510000 as shown in Table 19. Maximum total damaging rate which can be accepted is 0.05. Minimum acceptable late delivery is 0.15.

The multi objective linear program formulation is presented as min Z_1, max Z_2, Z_3.Then the linear membership function is used for fuzzifing the objective functions. The data set for the values of the lower bounds and upper bounds of the objective functions and a fuzzy number for the demand are given in Table 19.

Table 18: Supplier's quantitative information

Suppliers	Purchasing cost		Defects(rate)		On time delivery(rate)		Capacity		Price	
	j=1	j=2	j=1	j=2	j=1	j=2	j=1	j=2	j=1	j=2
1	65	64	0.03	0.05	0.15	0.18	90000	40000	4	2
2	45	48	0.04	0.01	0.09	0.06	85000	95000	2	4
3	50	56	0.03	0.02	0.17	0.07	65000	45000	4	4
4	54	60	0.04	0.02	0.15	0.1	50000	35000	5	5

Table 19: Data set for membership functions

	μ=0	μ=1	μ=0
Purchasing cost	-	26798779	30405000
Quality	14112.5	17562.5	-
Service	59600	62875	-
Demand	495000	500000	510000

Table 20: Optimal solution

$Max\ w_i \lambda_i$	Z=0.717954	λ_1 =0.776773	λ_2 =0.461568	λ_3 =0.960206
	X_{111}=71499.53	X_{211}=71498.41	X_{311}=65000	X_{411}=71499.71
	X_{121}=71500.56	X_{221}=71497.19	X_{321}=45000	X_{421}=35000

Optimal order quantities allocated to suppliers is done by fuzzy multi objective linear program which was integrated with TOPSIS, AHP improved by rough set theory and the optimal solution is presented in Table 20. From the table it is clear that the allocated order quantities to each supplier for a particular product in that period.

5. Conclusion:

In this paper a new algorithm is proposed for weighted additive fuzzy linear program. Weights of the objectives are obtained by rough set integrated with AHP and TOPSIS. Fuzzy constraint elements are in triangular form, to make it as a crisp form ranking method is used. If the triangular numbers are not in symmetrical form then this method can be used. Triangular number for the constraint helps to find a solution in an interval on the real line. This approach is able to help the DM's to evaluate the suppliers detail in a systematic manner without ambiguous, imprecision etc due to mathematical reasons in order to find out the appropriate order to each of them and allows purchasing managers to manage supply chain performance on service, cost, quality etc. The model has applied to professionally well manage company and the results were found out consistent and reliable Management feels that the method adopted is more realistic and promising as it involves the uncertainty in technological coefficients and it is comfortable to find an optimal solution in an interval. Managers are able to calculate the order quantity assigned to each supplier. Future research goal is to focus on other multi criteria decision making techniques integrated with rough set theory and it can applied to many fields such as machine learning, knowledge discovery, medical field, Project selection etc.

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