JOINT CHANNEL ESTIMATION AND DECODING OF RAPTOR CODE ON FADING CHANNEL

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Abstract

In this paper, the problem of transmission of Raptor codes over fading channel is considered. We present in this paper joint decoder architecture for Raptor codes over phase coherent fading channel. The proposed scheme does not require transmission of pilot symbols or extra coded bits for achieving the same BER performance. The receiver uses sum-product algorithm to jointly estimate the channel and decode the Raptor coded symbols. We also compare the performance of the practical decoder architecture proposed in the paper with existing architecture using LDPC codes which uses joint decoding.

Keywords: Raptor code, LDPC code, fading channel, channel estimation, belief-propagation.

1. Introduction

In the last decade, iterative decoding algorithm have revolutionized the field of error correction. Modern error correction codes like Turbo codes, low density parity check (LDPC) codes use iterative algorithm for their decoding. These codes employ iterative probabilistic algorithms for decoding and is considered to be among the most efficient decoding algorithm and perform very close to channel capacity limits.

LDPC codes are an important class of codes employing iterative decoding. It was shown in [1] that LDPC codes can approach, even outperform, the performance of turbo codes. Although LDPC codes and its iterative *a priori probability* (APP) decoder were invented by Gallager in 1960s, his work was largely forgotten for more than 30 years [2]. After the invention of Turbo codes, the interest in LDPC codes was rekindled in research community.

LDPC codes are represented through a sparse parity check matrix or a sparse bipartite graph consisting of two disjoint set of variable and check nodes. Iterative sum-product algorithm is implemented over such a bipartite graph for decoding LDPC codes over additive white Gaussian noise (AWGN) channel. At each iteration, the decoder compute the probability that a given variable node is 0 or 1, given the value of variable nodes in previous iterations.

Raptor codes are another class of codes which apply iterative sum-product algorithm, or iterative belief propagation algorithm, for decoding. Raptor codes have been developed as a class of rateless codes for reliable transmission over erasure channels with unknown capacity [3]. Raptor codes are rateless codes in the sense that code rate is not fixed a priori, that is, rate of the code can be adapted depending on the state of the channel.

In this paper we consider fixed rate Raptor code over block fading channels. Block fading channel model is widely used to model dispersive channels where the channel state remains fixed over a block of given length. Amount of fade may vary from block to block. For Rayleigh block fading channels, the channel states are multiplicative fading coefficients. We also extend our study of the performance of the proposed architecture to Jake's fading model. Performance of fixed rate Raptor code have been studied in [4] for various fading channels. It was shown that over memoryless fading channels Raptor codes provide performance that is comparable with or better than several state-of-the-art codes. It has been observed that Raptor codes significantly outperform turbo codes by 3.2-4.4dB. This proves the versatility and effectiveness of Raptor codes for wireless mobile communication. Compared to AWGN channel Raptor codes perform about 2 dB worse in Rayleigh fading channel. This gap can be reduced if knowledge of channel state information (CSI) is available with the decoder. Use of CSI have been in use for some time for improving the performance of channel codes over fading channels. Authors in [5] proposed a linear filter based channel estimation method using pilot symbols for decoding turbo codes over flat fading channels. Iterative estimation and decoding was implemented using hard and soft decision feedback. One drawback mentioned was the presence of error floor higher than turbo-coded system with ideal BPSK system. Another pilot based joint channel estimation and detection of LDPC codes in fading channel was proposed by Niu et al [6] for M-ary modulation. Jin et al [7] use an averaging method to estimate the fading amplitudes and them applying iterative sum-product algorithm. After each iteration, the value of the fading amplitude is re-estimated and the iteration continues till stopping condition is reached. A method to encode pilot bits together with information bits by using a systematic encoder is proposed in [8]. By simulation and analysis, authors have found that using encoded pilot BER performance can be improved at same complexity as that of conventional decoder with pilots.

In this paper, we propose a decoder for Raptor codes on block fading channel. We focus on Raptor codes as they are linear time encodable and decodable [3]. Also these codes do not exhibit error floor [10] and provide near Shannon capacity performance when used with fixed rates over memoryless symmetric channels. The proposed design is based on the architecture for decoding LDPC codes on fading channel by Jin et al [7]. As in [7], the decoder not require pilot symbols for estimating fading coefficient, thus saving energy required for transmission of pilot symbols. We simulate the performance of the proposed design over a phase coherent block fading channel. We also simulate the design for more realistic mobile communication channel, like Jake's model [9]. The rest of the paper is organized as follows. In section 2 we give the system model. Section 3 describes decoding algorithm of Raptor code using sum-product algorithm and the proposed architecture. Section 5 gives simulation results and finally section 6 is conclusion.

2. System Model

2.1. *Transmitter*

A block diagram model of the system under consideration is shown in figure 1. Let *C* be a binary linear LDPC code with dimension k and codeword length *n*. The random *source symbols* input to the LDPC pre-code encoder is a bipolar sequence $\{d_j\}$, $1 \leq j \leq k$. The LDPC encoded bit stream, called *input symbols* $\{v_i\}$ in the terminology of Raptor codes, is then coded by Luby Transform (LT) encoder [3]. It is the LT encoder which makes the code *rateless*, that is, LT encoder can produce an endless stream of symbols, which are then BPSK modulated (baseband equivalent) to produce $\{y_k\}$. The cascade of a pre-code (LDPC code) and LT code forms Raptor code with parameter $(L, C, \Omega(v))$. The parameter $\Omega(x) = \sum_t \Omega_t x^t$ is the generator polynomial of the LT code, where $\Omega_1, \Omega_2, \ldots, \Omega_{L'}$ is distribution on $\{1, \ldots, L'\}$. Ω_t denotes that probability that weight *t* is chosen. An integer *t* from $\{1, \ldots, L'\}$ is selected according to the distribution $\Omega(x)$ and *t* input symbols (output of LDPC code) are randomly selected and added modulo-2 to produce Raptor coded *output symbols*. In this paper we consider fixed length Raptor code of length *N*.

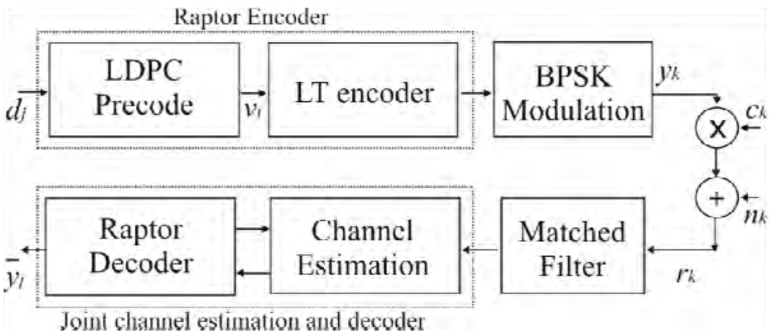

Fig 1. System block diagram of encoding and decoding of Raptor code over fading channel.

2.2. *Channel*

The transmitted signal is passed through a discrete-time slow fading channel with additive white Gaussian noise. The decision statistic after matched filtering is as follows:

$$r_k = c_k y_k + n_k \tag{1}$$

Here, $\{n_k\}$ is statistically independent complex Gaussian random variable with zero-mean and variance σ^2, and c_k is the fading associated with *k*-th sample with $E[c_k^2] = 1$. In this paper, we consider two types of fading channels, block fading and Jake's fading model. In block fading model it is assumed that the magnitude of fading coefficient c_k is constant for a block length of *h*. For Jake's isotopic scattering model [5], it has been assumed that real and isotopic parts of c_k are independent with autocorrelation

$$R_c[k] = \frac{1}{2} J_0(2\pi f_d T_s k) \tag{2}$$

where, f_d is maximum Doppler shift between transmitter and receiver, T_s is the symbol period, and $J_0(.)$ is the zeroth order Bessel function of first kind.

3. Joint Channel Estimation and Decoding

Displayed Fading of received signal amplitude will cause bit error rate to be more than the channel with no fading because variation in received signal strength causes fall in average signal to noise ratio (SNR). Probability of error can be reduced if channel state information is available with the decoder compared to the case where no CSI is available. Next we describe the relations which will lead to the equation for estimating fading coefficient of the channel.

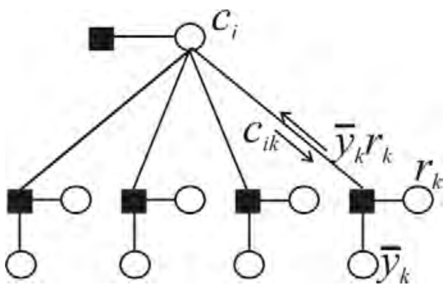

Fig 2. Factor graph of channel estimator

The minimum mean-square-error (MMSE) estimate of the channel gain is found as [5]

$$\hat{c}_k = \sum_{i=-\lfloor h/2 \rfloor}^{h/2} w_i y_{k-i} r_{k-i} \tag{3}$$

where h is the size of the estimation filter and w_i is the set of filter coefficients found by solving Wiener-Hopf equations. For slow fading ($f_d T_s \ll 1$) and filter length h sufficiently small ($h \ll (f_d T_s)^{-1}$), filter coefficients can be approximated as $w_i \approx 1/(h + 2\sigma^2)$. For $h \gg 2\sigma^2$, it can be further approximated as $w_i \approx 1/h$. Then the filter in (3) becomes a moving average filter

$$\hat{c}_k = \frac{1}{h} \sum_{i=-\lfloor h/2 \rfloor}^{h/2} y_{k-i} r_{k-i} \tag{4}$$

This relation is shown in the form of factor graph in Fig. 2 [6], [7], where circles represent variable nodes and filled squares represent factor nodes. In the figure $\bar{y}_k$ and r_k represents the hard-decision decoded symbol and received symbol, respectively. They are connected to *k*th *channel factor node*, which then updates *i*th *channel state node* with message $\bar{y}_k r_k$. The message from *i*th channel state node to *k*th channel factor node is c_{ik}. With the rules of factor graph and sum-product algorithm [13], and estimating the channel for each block of length *h* (instead of moving average), the message c_{ik} becomes [6]:

$$\hat{c}_{ik} = \frac{1}{h-1} \sum_{j=1, j \neq k}^{h} r_j \bar{y}_j \tag{5}$$

Thus we have an algorithm using factor graph for estimating the channel coefficient. Using this estimator and decoder for Raptor code, we can form an iterative receiver for decoding Raptor codes over slow fading channel. Factor graph for such an iterative receiver is shown in figure 3. It consists of channel estimator, LT decoder, and LDPC decoder in cascade. Channel output is received as r_k at the check node of channel estimator as shown in figure 2. Variable nodes r_k are not shown in figure 3 in order not to clutter the figure. We next describe the overall algorithm for decoding Raptor codes over fading channel with CSI iteratively estimated at the receiver.

1. Input to the LT code part of factor graph is $z_k,\ k = 1, \dots, N$, which is the LLR of *k*th output symbol received from the channel estimator. In the first iteration, when channel estimation is not available, z_k is simply the channel LLR given as $z_k = 4r_k/N_0$, where, $N_0 = \sigma^2/2$. For subsequent iterations, $z_k = 4r_k c_k/N_0$, where c_k has been found in channel estimate step in (5).

2. Updating rules for the LT decoder at *l*th iteration is given as follows. The message from output nodes of LT code (check nodes) to input nodes (variable nodes of LT code) is

$$m_{oi}^{l} = 2\ \mathrm{atanh}\left(\tanh\left(\frac{z_k}{2}\right) \prod_{i' \neq i} \tanh\left(\frac{m_{i'o}^{l}}{2}\right)\right), \quad o = 1, \dots, N \tag{6}$$

and in the return path, within LT code, the message from variable nodes to check nodes is

$$m_{io}^{l+1} = d^{LDPC}{}_{i}^{l} + \sum_{o' \neq o} m_{o'i}^{l}, \quad i = 1, \dots, n \tag{7}$$

In each iteration, message passed from LT code part of factor graph to LDPC code part of factor graph is denoted by d_i^{LT}. It is the sum of all messages coming to variable node *i*, given as

$$d_i^{LT^l} = \sum_{o \in N(i)} m_{oi}^l, \qquad i = 1, \ldots, n \tag{8}$$

where, $N(i)$ is the set of LT code check nodes neighboring the variable node *i*.

3. The update rule for the LDPC part of the factor graph are given by

$$m_{ic}^l = d^{LT^l}{}_i + \sum_{c' \neq c} m^{l-1}{}_{c'i}, \qquad i = 1, \ldots, n \tag{9}$$

and

$$m_{ci}^l = 2\,\mathrm{atanh}\left(\prod_{i' \neq i} \tanh\left(\frac{m_{i'c}^l}{2}\right)\right), \qquad c = 1, \ldots, n-k. \tag{10}$$

The message $d^{LDPC^l}{}_i$ is the sum of all LLR arriving the variable node *i* of LDPC code. If N(i) is the set of all check nodes of LDPC code connected to variable node *i* of LDPC code, then

$$d^{LDPC^l}{}_i = \sum_{c \in N(i)} m^{l-1}{}_{ci}, \qquad i = 1, \ldots, n \tag{11}$$

4. Thus the path of the message is as follows: check nodes of LT code - variable nodes of LT codes - variable nodes of LDPC code - check node of LDPC code – variable nodes of LDPC code – variable nodes of LT code and back again to check nodes of LT code. At this point, after each such iteration, source symbol is hard decoded as

$$\bar{y}_k = \begin{cases} 1 & if\ x_k \geq 0 \\ -1 & if\ x_k < 0 \end{cases} \tag{12}$$

where, x_k is an LLR computed after each iteration as

$$x_k = z_k + 2\,\mathrm{atanh}\left(\prod_i \tanh\left(\frac{m_{io}^l}{2}\right)\right), \qquad o = 1, \ldots, N \tag{13}$$

5. Using the hard decision obtained in (12), the channel fading coefficients are re-estimaetd using (5) in channel estimator block in Fig. 3. If number of iterations is less than maximum number of iterations, go back to step 1. Otherwise, quit the loop with decoded output as $(1 - \bar{y}_k)/2 \in \{0,1\}$.

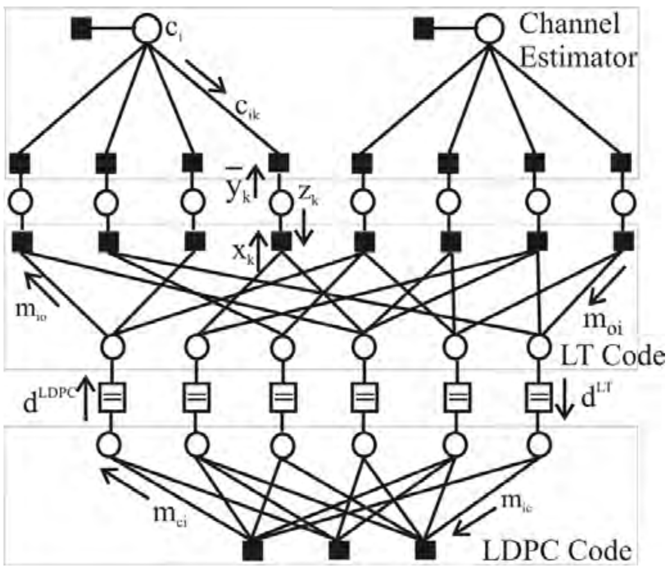

Fig 3. Factor graph of iterative receiver for Raptor code over fading channel. Channel input r_k is not shown in the figure for clarity.

4. Simulation Results

In this section we describe the simulation results obtained using the proposed architecture. In both the simulations, we consider a code word length N of 20000 and message length k of 9801. The LT code and LDPC code degree distribution are taken from [14] without considering any optimization method. The LT degree distribution is $\Omega(x) = 0.008x + 0.494x^2 + 0.166x^3 + 0.073x^4 + 0.083x^5 + 0.056x^8 + 0.037x^9 + 0.056x^{19} + 0.025x^{65} + 0.003x^{66}$. The LDPC code is regular (2,100) code of rate 0.98. For both channel cases, 100 iterations were run for obtaining the decoded symbols.

Figures 4 and 5 show the simulation results for block fading channel and Jakes's fading model, respectively. For each model we consider three scenarios, (i) perfect CSI at the receiver, (ii) CSI estimated by the receiver using the proposed method, (iii) no CSI available at the receiver. We mention for comparison purpose that thresholds for length 10^6 length, rate 1/2, unoptimized regular (3,6) LDPC codes on the AWGN channel, the fading channel with CSI, and fading channel with no CSI, are 1.10 dB, 3.06 dB, and 4.06 dB, respectively [12].

For block fading channel, similar to [7], we have taken block length h = 20 within which fading value remains constant. The threshold value for estimated CSI case found by simulation in [7] for codeword length of 200,000 was about 3.5 dB. We obtain similar value of threshold with Raptor code, but with much smaller codeword length, as shown in fig 4. This makes Raptor code a better alternative for wireless communication, as end-to-end delay for codes with larger block length is higher. The same decoder was applied for decoding Raptor code in Jake's fading model for $T_s f_d = 0.005$. With this parameter, the channel fading can be considered slow and the assumption that fading coefficient remains constant for block length of h holds true. Simulation results are shown in figure 5. We find that performance of the proposed decoder with Jake's fading model is better than block fading case in fig 4 and with LDPC code in [7]. It is because the variation of channel coefficients is much smaller in consecutive blocks, compared to block fading model. In block fading model, the channel coefficients vary randomly in consecutive blocks and are Rayleigh distributed.

5. Conclusion

In this paper we proposed decoder architecture for Raptor codes on fading channels. The decoder does not require pilot symbols for estimating channel fading coefficients, with only assumption that fading coefficients do not change much in a block period. We find that with Raptor code, capacity approaching performance on fading channel can be achieved with much smaller block length compared to decoder with similar architecture using LDPC codes. The decoder performs even better in Jake's slow fading channel. Our future work is to optimize Raptor codes for fading channels and derive the threshold for these codes using density evolution analysis.

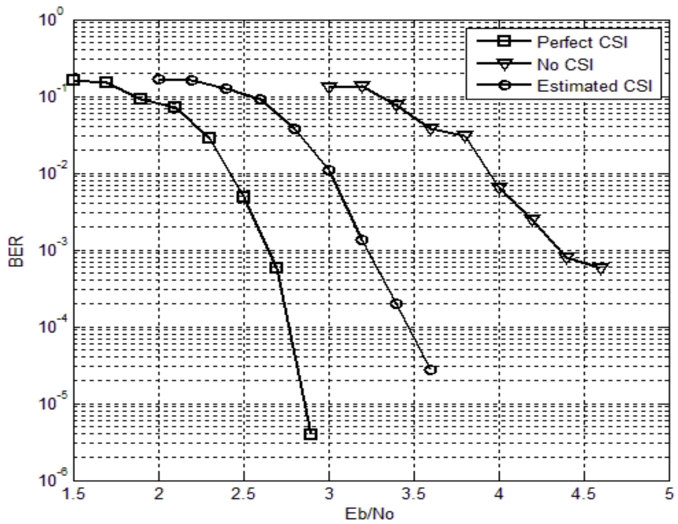

Fig 4. Raptor coded bit error performance versus E_b/N_0 over block fading channel for block length of $h = 20$ and with CSI estimated iteratively at the decoder. Figure also shows the BER plot with perfect CSI and no CSI available at the decoder.

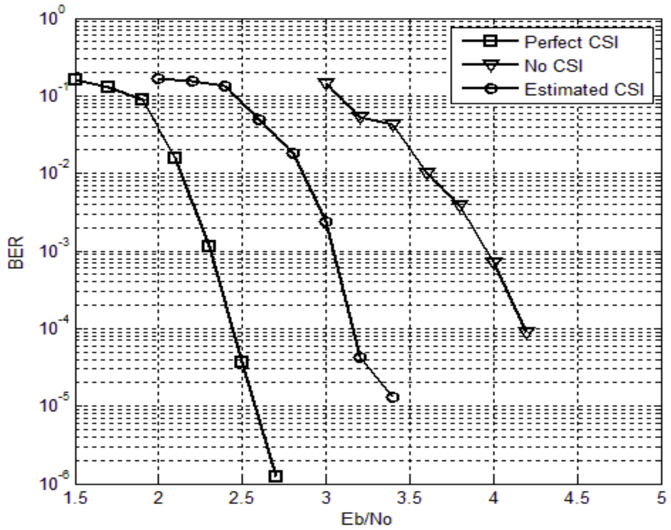

Fig 5. Raptor coded bit error performance versus E_b/N_0 over Jake's fading channel and estimation filter block length of $h = 20$ at the receiver.

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