I-V IF SHORTEST PATH IN A MULTIGRAPH

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Abstract

The theory of multigraphs is a generalization of the Theory of graphs. The existing algorithms to find the fuzzy shortest path or intuitionistic fuzzy shortest paths in graphs are not applicable to multigraphs. In this paper we develop a method to search for an i-v intuitionistic fuzzy shortest path in a directed multigraph and then develop, as a special case, a fuzzy shortest path in a multigraph. We coin the concept of classical Dijkstra's algorithm which is applicable to graphs with crisp weights, and then extend this concept to multigraphs where the weights of the arcs are i-v IFNs. It is claimed that the method might play a significant role in several application areas of computing, communication network, transportation systems, etc. particularly in those networks that cannot be sculptured into graphs however into multigraphs.

***Keywords*:** Multigraphs, i-v IFS, i-v IFN, i-v IF-Min-Weight arc-set, i-v IF shortest path estimate, i-v IF relaxation.

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1. Introduction

Graph theory [1-5] has wide varieties of applications in several branches of Engineering, Science, Social Science, Medical Science, Economics, etc. to list a few only out of many. Multigraph [6-18] is a generalized concept of graph where multiple edges (or arcs/links) may exist between nodes. For example, in a communication model in a MANET, multipath features are very common. Two neighboring routers in an exceedingly constellation would possibly share over multiple direct connections between them (instead of simply one), thus on cut back the bandwidth as compared if one association is employed. Many real world things of communication network, transportation network, etc. can't be sculptured into graphs, however are often well sculptured into multigraphs. In many of these directed multigraphs, the weights of the arcs are not always crisp or fuzzy but intuitionistic fuzzy and in more reality i-v intuitionistic fuzzy. Throughout in this research article, the work has been carried out in multigraphs without loops.

One of the first studies on fuzzy shortest path problem (FSPP) in graphs was done by Dubois and Prade [19] and then by Klein [20]. However, few more solutions to FSPP proposed in [21-30] are also interesting. Though the work of Dubois and Prade [19] was a major break-through, but that paper lacked any practical interpretation as even if fuzzy shortest path is found, but still this may not actually be any of the path in the corresponding network for which it was found. The concept of intuitionistic fuzzy graphs was initiated and studied by Atanassov in [5-12]. Intuitionistic fuzzy multigraphs were also studied by Atanassov in [5-12]. In [31] Sathi developed an interesting method to find in intuitionistic fuzzy shortest path in a graph. But, in this paper we solve this problem to find i-v intuitionistic fuzzy shortest path in a multigraph where the arc-weights are i-v intuitionistic fuzzy numbers (i-v IFNs), and then we reduce the method to the case of finding fuzzy shortest path in a multigraph.

We follow the concept of classical Dijkstra's algorithm for graphs and extend this concept to multigraphs where the weights of the arcs are i-v IFNs. Earlier in [32-45] we made a generalization of Dijkstra's Algorithm for a directed multigraph, all data being crisp neither fuzzy nor intuitionistic fuzzy. Then we proposed a fuzzy shortest path in a directed multigraph. In [46-57] we introduced a new mathematical modeling in the theory of multigraphs by incorporating variable costs with respect to real time information. Then we considered an intuitionistic fuzzy 'real time' multigraph where we presented a theoretical mathematical modeling only.

2. Preliminaries

In this section we present a basic preliminaries of the IFS theory of Atanassov and then a brief note about multigraphs.

2.1. *Intuitionistic fuzzy set (IFS)*

The Intuitionistic fuzzy set (IFS) theory of Atanassov [5-8] is now a well known powerful soft computing tool to the world. If X be a universe of discourse, an intuitionistic fuzzy set A in X is a set of ordered triplets A = $\{ < x, \mu_A(x), \upsilon_A(x) >: x \in X \}$ where $\mu_A, \upsilon_A : X \to [0, 1]$ are functions such that $0 \leq \mu_A(x) + \upsilon_A(x) \leq 1 \ \forall x \in X$. For each $x \in X$ the values $\mu_A(x)$ and $\upsilon_A(x)$ represent the degree of membership and degree of non-membership of the element x to $A \subset X$, respectively, and the amount $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ is called the hesitation part. Of course, a fuzzy set is a particular case of the intuitionistic fuzzy set if $\pi_A(x) = 0, \forall x \in X$. For details of the classical notion of intuitionistic fuzzy set (IFS) theory, one could see the books authored by Atanassov [6,7]. The concept of an intuitionistic fuzzy number (IFN) is of importance for quantifying an ill-known quantity. In our work here throughout, we use the notion of triangular intuitionistic fuzzy numbers(IFN) and the basic operations like IF addition $\oplus$, IF subtraction $\ominus$, and 'ranking' of intuitionistic fuzzy numbers (IFNs) etc.

Trivially, any crisp real number can be viewed as a fuzzy number or as an IFN. There is no unique method for ranking n number of IFNs, because all the existing methods [15,17,22] are soft computing methods. Each method has got merits and demerits depending upon the properties of the application domains and the problem under consideration. However, if $A_1, A_2, A_3, \ldots, A_n$ be n IFNs sorted in IF ascending order (in fact it is a kind of non-ascending order, assuming that the IF equal IFNs takes corresponding positions at random if there is no loss of generality) by any good pre-decided method i.e. if $A_1 \prec A_2 \prec A_3 \prec \ldots \prec A_n$ then A_1 and A_n are called respectively the **IF-min** and **IF-max** of these n IFNs. Almost all the existing methods [15,17,22] of ranking IFNs were developed independently i.e. not as extensions of the existing methods [1,2,16,20,31,33,35] of ranking of fuzzy numbers [32]. Although several authors [1,2,16,20,31,33,35] have reported several ranking methods of fuzzy numbers, all are having limitations too i.e. not an absolute method suitable for every application domain. However, if $A_1, A_2, A_3, \ldots, A_n$ be n fuzzy numbers sorted in fuzzy ascending order by a pre-decided method i.e. $A_1 \prec A_2 \prec A_3 \prec \ldots \prec A_n$, then A_1 and A_n are called respectively the fuzzy-min and fuzzy-max of these n fuzzy numbers.

2.2. *Multigraph*

A multigraph A is an ordered pair (V, E) which consists of two sets V and E, where V or V(A) is the set of vertices (or, nodes), and E or E(A) is the set of edges (or, arcs). Here, although multiple edges or arcs might exist between pair of vertices but we consider that no loop exists. Multigraphs may be of two types: undirected multigraphs and directed multigraphs. In an undirected multigraph the edge (i, j) and the edge (j, i), if exist, are obviously identical unlike in the case of directed multigraph. For a latest algebraic study on the theory of multigraphs, the work [38-53] may be seen. Figure 1 shows below a directed multigraph A = (V, E), where V = {P, Q, R, S, T} and E = {PQ_1, PQ_2, PR, RP, PS, RS, SQ, ST, RT, QT, TQ}.

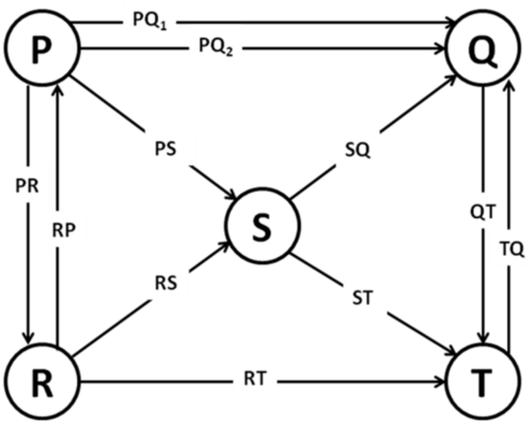

Fig 1. Multigraph A

A multigraph B = (W, F) is called a submultigraph of the multigraph A = (V, E) if W ⊆ V and F ⊆ E. The Figure 2 shows a submultigraph B of the multigraph A of Figure 1.

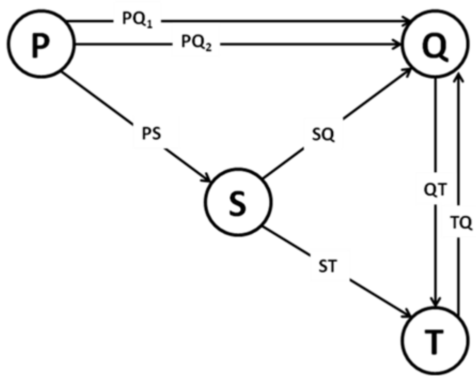

Fig 2. Submultigraph B of the multigraph A

In most of the real life problems of networks, be it in a communication model or transportation model, the weights of the arcs are not always crisp but i-v intuitionistic fuzzy numbers (or, at best fuzzy numbers). For example, the Figure 3 below shows a public road transportation model for a traveler where the cost parameter for traveling each arc have been available to him as an I-V IFN. I-V IFNs are the more generalized form of fuzzy numbers involving two independently estimated degrees: degree of acceptance and a degree of rejection.

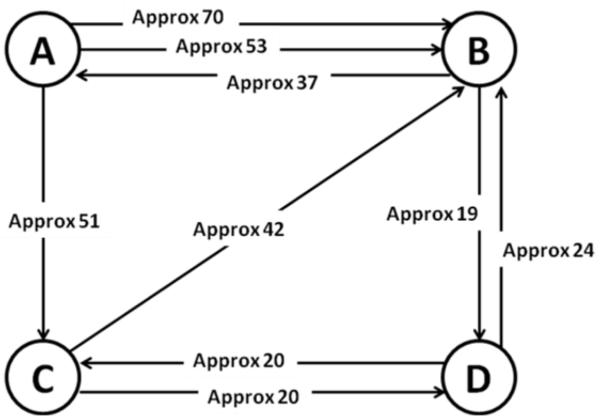

Fig 3. Multigraph G with I-V IF weights of arcs.

In this paper we consider such type of real situations in multigraphs of communication systems or transportation systems and develop a method to find an i-v intuitionistic fuzzy shortest path from a source node to a destination node.

3. I-V IF Shortest Path in a Multigraph

Sathi [34] used a heuristic methodology for solving the IF shortest path problem in a graph using the Intuitionistic Fuzzy Hybrid Geometric (IFHG) operator, with the philosophy of Dijkstra's Algorithm. The present work here is mainly motivated by the problem posed in [34] by Sathi, but in a multigraph. In [28], M. G. Karunambigai, Parvathi Rangasamy, N. Palaniappan in a team work with Atanassov, presented a model based on dynamic programming to find the shortest paths in intuitionistic fuzzy graphs. A. Nagoor Gani in [4] also developed a method on searching intuitionistic fuzzy shortest path in a network. But the work in this paper is on a multigraph, and not developed as a generalization of any previous work.

In our method here, we solve SPP for multigraphs where we also use the notion of Dijkstra's Algorithm but with simple soft-computations without using any hybrid geometric operators, using only basics of Atanassov's IFS operators [6]. For this, first of all we need to define the terms : I-V IF-Min-Weight arc-set, I-V IF shortest path estimate (d[v]) of a vertex, I-V IF relaxation of an arc, etc. in the context of the multigraphs, and develop few sub algorithms.

3.1. I-V IF-Min Weight Arc-set of a Directed Multigraph

Consider a directed multigraph G where the arcs are of I-V IF weights. Suppose that there are n number of arcs from the vertex u to the vertex v in G, where n is a non-negative integer. Let W_{uv} be the set whose elements are the arcs between vertex u and vertex v, but keyed & sorted in non-descending order by the value of their respective I-V IF weights, using a suitable pre-decided ranking method of I-V IFNs.

$\therefore \quad W_{uv} = \{ (uv_1, w_{1uv}), (uv_2, w_{2uv}), (uv_3, w_{3uv}), \ldots.., (uv_n, w_{nuv}) \}$
where, uv_i is the arc-i from vertex u to vertex v and w_i is the I-V IF weight of it, for $i = 1, 2, 3, \ldots, n$.

If two or more number of I-V IF weights are equal then they may appear at random at the corresponding place of non-descending array with no loss of generality in our discussion.

If there is no confusion, let us denote the multiset $\{ w_{1uv}, w_{2uv}, w_{3uv}, \ldots\ldots, w_{nuv} \}$ also by the same name W_{uv}. Let w_{uv} be the I-V IF-min value of the multiset $W_{uv} = \{ w_{1uv}, w_{2uv}, w_{3uv}, \ldots\ldots, w_{nuv} \}$.
Clearly, $w_{uv} = w_{1uv}$, as the multiset W_{uv} is already sorted.

Then the set $W = \{ < (u,v), w_{uv} > : (u,v) \in E \}$ is called the I-V IF-Min-weight arc-set of the multigraph G. Suppose that the subalgorithm I-V IFMWA (G) returns the I-V IF-Min-weight arc-set W.

3.2. I-V IF Shortest path estimate d[v] of a vertex v in a directed multigraph

Suppose that s is the source vertex and the currently traversed vertex is u. There is no single value of weight for arc between vertex u and a neighbor vertex v, rather there are multiple value of weights as there are multiple arcs between vertex u and vertex v. Using the value of w_{uv} from the I-V IF-min weight multiset w of a directed multigraph, we can now find the I-V IF shortest path estimate i.e. d[v] of any vertex v, in a directed multigraph as below (see Figure 4) :-

(I-V IF shortest path estimate of vertex v) = (I-V IF shortest path estimate of vertex u) ⊕ (I-V IF-Min of all the I-V IF weights corresponding to the arcs from the vertex u to the vertex v)**.**

or, $d[v] = d[u] \oplus w_{uv}$.

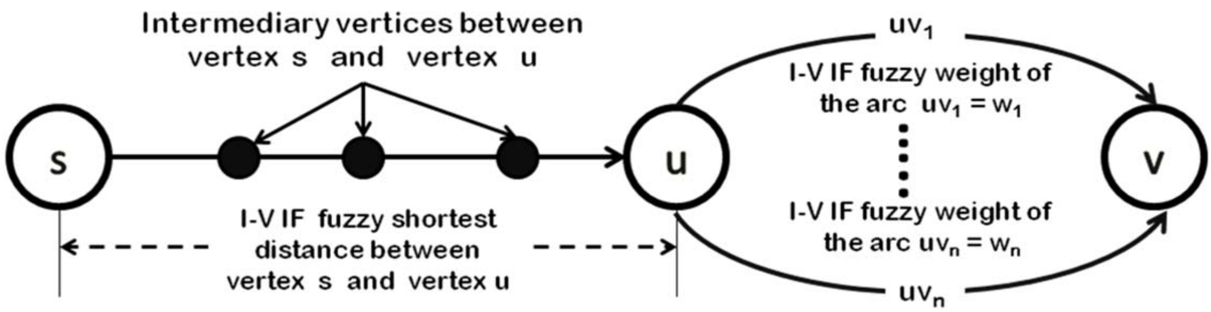

Fig 4. Estimation procedure for d[v]

3.3. I-V IF Relaxation of an arc

We extend the classical notion of relaxation to the case of I-V IF weighted arcs. By I-V IF relaxation we shall mean here the relaxation process of an arc whose weight is an I-V IFN. For this, first of all we initialize the multigraph along with its starting vertex and I-V IF shortest path estimate for each vertices of the multigraph G.

The following ‘I-V IF-INITIALIZATION-SINGLE-SOURCE’ algorithm will do :

I-V IF-INITIALIZATION-SINGLE-SOURCE (G, s)
1. For each vertex $v \in V[G]$
2. $d[v] = \infty$
3. $v.\pi = NIL$
4. $d[s] = 0$

After the I-V IF initialization, the process of I-V IF relaxation of each arc begins. The sub-algorithm I-V IF-RELAX, plays the vital role to update d[v] i.e. the I-V IF shortest distance value between the starting vertex s and the vertex v (which is neighbor of the current traversed vertex u). See the corresponding Figure 5.

I-V IF-RELAX (u, v, W)
1. IF $d[v] \succ d[u] \oplus w_{uv}$
2. THEN $d[v] \leftarrow d[u] \oplus w_{uv}$
3. $v.\pi \leftarrow u$

where, $w_{uv} \in W$ is the I-V IF-Min weight of the arcs from vertex u to vertex v, and $v.\pi$ denotes the parent node of vertex v.

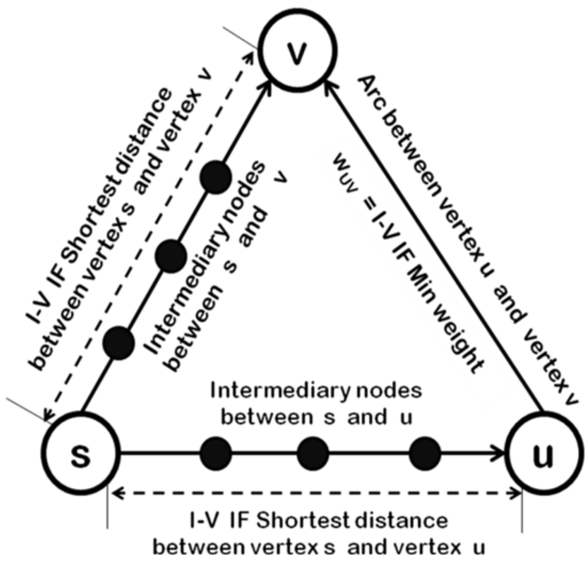

Fig 5. Diagram showing how the I-V IF-RELAX algorithm works

3.4. I-V IF Shortest Path Algorithm (I-V IFSP Algo)

In this section we now present our main algorithm to find single source I-V IF shortest path in a multigraph. We name this i-v intuitionistic Fuzzy Shortest Path Algorithm by the title I-V IFSPA. In this algorithm we use the above sub-algorithms, and also the sub-algorithm EXTRACT-I-V IFMIN(Q) which extracts the node u with minimum key using I-V IF ranking method and updates Q.

I-V IFSPA (G, s)
1. I-V IF-INITIALIZATION-SINGLE-SOURCE (G, s)
2. $W \leftarrow$ I-V IFMWA (G)
3. $S \leftarrow \emptyset$
4. $Q \leftarrow V[G]$
5. WHILE $Q \neq \emptyset$
6. DO $u \leftarrow$ EXTRACT-I-V IFMIN (Q)
7. $S \leftarrow S \cup \{u\}$
8. FOR each vertex $v \in Adj[u]$
9. DO I-V IF-RELAX (u, v, W)

Example 3.1.

Consider the following directed Multigraph G with I-V IF weights (in Fig 6). We want to solve the single-source I-V IF shortest paths problem taking the vertex A as the source vertex and the vertex D as the destination vertex.

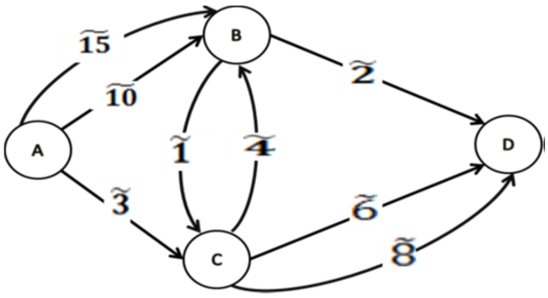

Fig. 6. Multigraph G

Let us see how the I-V IFSPA algorithm computes here.

$W_{AB} = \{(AB, \widetilde{15}), (AB, \widetilde{10})\} \Rightarrow w_{AB} = \widetilde{10}$
$W_{AC} = \{(AC, \tilde{3})\} \Rightarrow w_{AC} = \tilde{3}$
$W_{BC} = \{(BC, \tilde{1})\} \Rightarrow w_{BC} = \tilde{1}$
$W_{CB} = \{(CB, \tilde{4})\} \Rightarrow w_{CB} = \tilde{4}$
$W_{BD} = \{(BD, \tilde{2})\} \Rightarrow w_{BD} = \tilde{2}$
$W_{CD} = \{(CD, \tilde{6}), (CD, \tilde{8})\} \Rightarrow w_{CD} = \tilde{6}$

Therefore, in step-2, I-V IF MWA(G) returns W as below:-

$W_{AB} = \{(AB, \widetilde{10}), (AC, \tilde{3}), (BC, \tilde{1}), (CB, \tilde{4}), (BD, \tilde{2}), (CD, \tilde{6})\}$

Initially by Step-3 and 4,

$Q = \{A, B, C, D\}$ and $S = \Phi$.

Step- 6 to 9 yield (by step-5 i.e. satisfying the WHILE condition)

$Q \neq \Phi$ is TRUE

$$\begin{cases} u = A \\ S = S \cup \{A\} = \{A\} \text{ i.e. } S : A \\ Q = \{B, C, D\} \end{cases}$$

$Q \neq \Phi$ is TRUE

$$\begin{cases} u = C \\ S = S \cup \{C\} = \{A, C\} \text{ i.e. } S : A \rightarrow C \\ Q = \{B, D\} \end{cases}$$

$Q \neq \Phi$ is TRUE

$$\begin{cases} u = B \\ S = S \cup \{B\} = \{A, C, B\} \text{ i.e. } S : A \rightarrow C \rightarrow B \\ Q = \{D\} \end{cases}$$

$Q \neq \Phi$ is TRUE

$$\begin{cases} u = D \\ S = S \cup \{D\} = \{A, C, B, D\} \text{ i.e. } S : A \rightarrow C \rightarrow B \rightarrow D \\ Q = \{B, C, D\} \end{cases}$$

Since the condition '$Q \neq \Phi$' is now FALSE, the I-V IFSPA algorithm terminates yielding the following results:

The I-V IF shortest path from the source vertex A is : $A \rightarrow C \rightarrow B \rightarrow D$

And the d–values i.e. I-V IF shortest distance estimate-values of each vertex from the starting vertex A is **:**
$d[A] = 0$, $d[C] =$ I-V IFN $\tilde{3}$, $d[B] =$ I-V IFN $\tilde{7}$, $d[D] =$ I-V IFN $\tilde{9}$

4. Conclusion

Multigraph is a generalization of graph. There are many real life problems of network, transportation, communication, circuit systems, etc. which cannot be modeled into graphs but into multigraphs only. In many of these directed multigraphs, the weights of the arcs are not always crisp but i-v intuitionistic fuzzy (or fuzzy). In this paper we have developed a method to find I-V IF shortest path from a source vertex to a destination vertex of a directed multigraph.

As a special case, the method reduces to the method of finding fuzzy shortest path in a directed multigraph. There are some good methods [34] developed by various authors to solve IFSPP in graphs, but in this work we have solved I-V IFSPP in multigraphs.

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