Stochastic Modelling of a Computer System with Software Redundancy and Priority to Hardware Repair

V.J. Munday*
Department of Statistics, Ramjas College, University of Delhi, Delhi – 110007 (India)
Email: vjmunday@rediffmail.com

Permila
Department of Statistics, M.D. University, Rohtak – 124001 (India)
Email: permilasingh88@gmail.com

S.C. Malik
Department of Statistics, M.D. University, Rohtak – 124001 (India)
Email: sc_malik@rediffmail.com

Abstract

The idea of this paper is to determine reliability and economic measures of a computer system by providing software component as spare in cold standby and priority to hardware repair over software up-gradation. The hardware and software may fail independently from normal mode. There is a single server who is called immediately to conduct hardware repair and software up-gradation as and when required. The repair and up-gradation activities performed by the server are perfect. The random variables associated with time to failure, hardware repair and software up-gradation are statistically independent. The time to hardware and software failures follows negative exponential distribution, whereas the distributions of hardware repair and software up-gradation times are taken as arbitrary with different probability density functions. The expressions for various reliability and economic measures are derived in steady state using semi-Markov process and regenerative point technique. The trends of some important measures of system effectiveness have been observed for arbitrary values of the parameters and costs. The profit of the present model has also been compared with that of the model Munday and Malik [2015].

Keywords: Stochastic Modelling, Computer System, Software Redundancy, Priority to Hardware Repair and Reliability Measures.

1. Introduction

The stochastic modelling of a computer system has been done in the most of the papers by providing component wise redundancy in cold standby with the concept of priority in repair disciplines. Welke et al. [1995] have discussed reliability modeling of a hardware/software system. Also, Yadavalli et al. [2004] and Kumar et al. [2012] analyzed repairable system models using unit wise redundancy. But the concept of priority to hardware repair over software up-gradation has not been studied by the researchers so far in case of computer systems with component wise redundancy. However, Anand and Malik [2012] have tried to obtain reliability measures of a computer system with unit wise redundancy in cold standby and priority to hardware repair activities over software replacement. Barak and Malik [2014] and Kumar et al. [2015] analyzed system models with cold standby redundancy under different failures and repair policies with the concept of priority. Also, Munday and Malik [2015] tried to establish a stochastic model for a computer system by providing software redundancy in cold standby.

The main focus of the present paper on to determine reliability measures of a computer system by introducing software redundancy in cold standby with priority to hardware repair over software up-gradation. For this purpose, a stochastic model can be developed for a computer system by providing software redundancy. The hardware and software may fail independently from normal mode. There is a single server who is called immediately to conduct hardware repair and software up-gradation as and when required. The repair and up-gradation activities performed by the server are perfect. The random variables associated with time to failure, hardware repair and software up-gradation are statistically independent. The time to hardware and software failures follows negative exponential distribution, whereas the distributions of hardware repair and software up-gradation times are taken as arbitrary with different probability density functions. The expressions for various reliability and economic measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to hardware repair and software up-gradation, expected number of hardware repairs and software up-gradations and profit function are derived in steady state using semi-Markov process and regenerative point technique. The trends of some important measures of system effectiveness have been observed for arbitrary values of the parameters and costs. The profit of the present model has also been compared with that of the model Munday and Malik (2015).

2. Notations and Abbreviations

O	:	Computer system is operative
Scs	:	Software is in cold standby
a/b	:	Probability that the system has hardware / software failure
λ_1/ λ_2	:	Hardware/Software failure rate
HFUr /HFWr	:	The hardware is failed and under/waiting for repair
SFUg/SFWUg	:	The software is failed and under/waiting for up-gradation
HFUR/HFWR	:	The hardware is failed and continuously under/ waiting for repair from previous state
SFUG/SFWUG	:	The software is failed and continuously under /waiting for up- gradation from previous state
g(t)/G(t)	:	pdf/cdf of hardware repair time
f(t)/F(t)	:	pdf/cdf of software up-gradation time
$q_{ij}(t)$/ $Q_{ij}(t)$	:	pdf / cdf of first passage time from regenerative state S_i to a regenerative state S_j or to a failed state S_j without visiting any other regenerative state in (0, t]
$q_{ij.k}$ (t)/$Q_{ij.k}(t)$	:	pdf/cdf of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state S_k once in (0, t]
$M_i(t)$	:	Probability that the system up initially in state $S_i \in$ E is up at time t without visiting to any regenerative state
$W_i(t)$	:	Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
μ_i	:	The mean sojourn time in state S_i which is given by $\mu_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij}$, where T denotes the time to system failure.
m_{ij}	:	Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*'}(0)$
Ⓢ/©	:	Symbol for Laplace-Stieltjes convolution/Laplace convolution
*/**	:	Symbol for Laplace Transformation (LT)/Laplace Stieltjes Transformation (LST)
$\phi_i(t)$	:	cdf of first passage time from regenerative state S_i to a failed state
A_i (t)	:	Probability that the system is in up-state at instant't' given that the system entered regenerative state S_i at t=0.
$B_i^H(t)$	:	Probability that the server is busy in repairing the unit due to hardware failure at an instant 't' given that the system entered state S_i at t = 0.

$B_i^S(t)$	:	Probability that the server is busy due to replacement of the software at an instant 't' given that the system entered the regenerative state S_i at t = 0.
$NHR_i(t)$	:	Expected number of hardware repairs by the server in (0, t] given that the system entered the regenerative state S_i at t = 0.
$NSU_i(t)$	:	Expected number of software up-gradations in (0, t] given that the system entered the regenerative state S_i at t = 0.
K_0	:	Revenue per unit up – time of the system
K_1	:	Cost per unit time for which server is busy due hardware repair
K_2	:	Cost per unit time for which server is busy due software up – gradation
K_3	:	Cost per unit repair of the failed hardware
K_4	:	Cost per unit up-gradation of the failed software
P1	:	Profit of the system model Munday and Malik (2015)

3. Analysis of the System Model

The state transition diagram is shown in the following figure:

State Transition Diagram

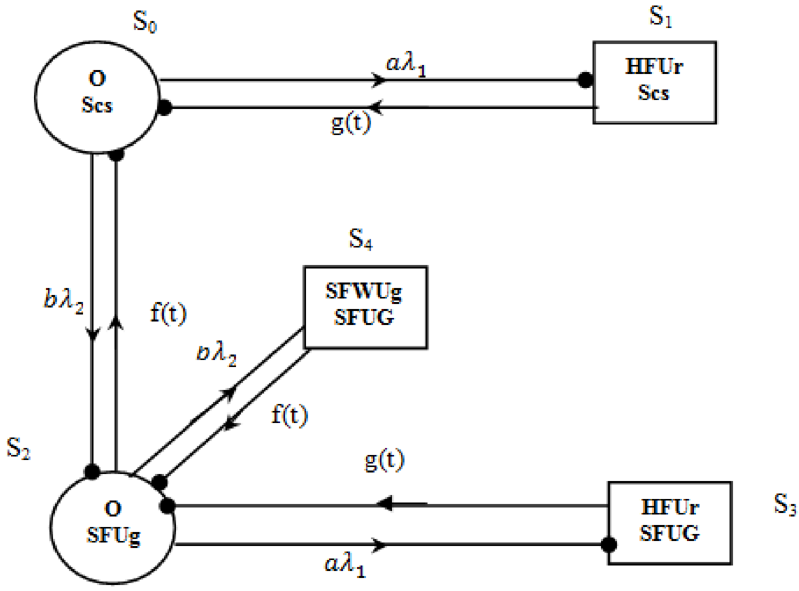

Fig. 1

○ Up-State □ Failed State ● Regenerative Point

3.1 Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements.

$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$

$p_{01} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}$, $\quad p_{02} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}$,

$p_{10} = g^*(0)$ $\quad p_{20} = f^*(a\lambda_1 + b\lambda_2)$

$p_{23} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}\{1 - f^*(a\lambda_1 + b\lambda_2)\}$ $\quad p_{24} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}\{1 - f^*(a\lambda_1 + b\lambda_2)\}$,

$p_{32} = g^*(0)$ $\quad p_{42} = f^*(0)$ (1)

For $g(t) = \alpha e^{-\alpha t}$ and $f(t) = \theta e^{-\theta t}$ we have

$p_{22.4} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}\{1 - f^*(a\lambda_1 + b\lambda_2)\}f^*(0)$

But, $f^*(0) = g^*(0) = 1$ and $p + q = 1$

It can be easily verified that

$p_{01} + p_{02} = p_{10} = p_{20} + p_{23} + p_{24} = p_{32} = p_{42} = p_{20} + p_{23} + p_{22.4} = 1$ (2)

The mean sojourn times (μ_i) in the state S_i are given by

$\mu_0 = \frac{1}{a\lambda_1 + b\lambda_2}$ $\quad\mu_1 = \frac{1}{\alpha}$

$\mu_2 = \frac{1}{a\lambda_1 + b\lambda_2 + \theta}$ $\quad\mu_3 = \frac{1}{\alpha}$ $\quad\mu_2' = \frac{(b\lambda_2 + \theta)}{\theta(a\lambda_1 + b\lambda_2 + \theta)}$

Also

$m_{01} + m_{02} = \mu_0, \quad m_{10} = \mu_1, \; m_{20} + m_{23} + m_{24} = \mu_2$

And $m_{20} + m_{21.3} + m_{22.4} = \mu_2'$ (3)

3.2 Reliability and Mean Time to System Failure (MTSF)

The expressions for $\phi_i(t)$ in terms of $Q_{ij}(t)$ are as follows:

$\phi_0(t) = Q_{02}(t) \,Ⓢ\, \phi_2(t) + Q_{01}(t)$

$\phi_2(t) = Q_{20}(t) \,Ⓢ\, \phi_0(t) + Q_{23}(t) + Q_{24}(t)$ (4)

Taking LST of above expressions (4) and solving for $\phi_0^{**}(s)$, we have

$$R^*(s) = \frac{1 - \phi_0^{**}(s)}{s}$$

By taking Laplace inverse transform of the above result, the reliability of the system model can be obtained. The MTSF is given by

$$MTSF = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N_1}{D_1} \quad (5)$$

Where $N_1 = \mu_0 + p_{02}\mu_2 \; and \; D_1 = 1 - p_{02}p_{20}$ (6)

3.3 Steady State Availability

The expressions for A_i (t) in terms of transition probabilities are given as:

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)© A_2(t)$

$A_1(t) = q_{10}(t)©A_0(t)$

$A_2(t) = M_2(t) + q_{20}(t)©A_0(t) + q_{22.4}(t)© A_2(t) + q_{23}(t)© A_3(t)$

$A_3(t) = q_{32}(t)©A_2(t)$ (7)

where $M_0(t) = e^{-(a\lambda_1 + b\lambda_2)t} \; and \; M_2(t) = e^{-(a\lambda_1 + b\lambda_2)t}\, \overline{F(t)}$

Taking LT of expressions (7) and solving for $A_0^*(s)$, the steady state availability is given by

$A_0(\infty) = \lim_{s \to 0} s\, A_0^*(s) = \frac{N_2}{D_2}$ (8)

Where $N_2 = p_{20}\mu_0 + p_{02}\mu_2$ and $D_2 = p_{20}\mu_0 + p_{20}p_{01}\mu_1 + p_{02}\mu_2' + p_{02}p_{23}\mu_3$ (9)

3.4 Busy Period of the Server

(a). *Due to Hardware Repair*

The expressions for $B_i^H(t)$ in terms of transition probabilities are as follows:

$B_0^H(t) = q_{01}(t)©B_1^H(t) + \; q_{02}(t)©B_2^H(t)$

$B_1^H(t) = W_1^H(t) + q_{10}(t)©B_0^H(t)$

$B_2^H(t) = q_{20}(t)©B_0^H(t) + q_{22.4}(t)©B_2^H(t) + q_{23}(t)©B_3^H(t)$

$B_3^H(t) = q_{32}(t)©B_2^H(t)$ (10)

where $W_1^H(t) = \; \overline{G(t)}\, dt$

(b). *Due to software Up-gradation*

The expressions for B_i^S (t) in terms of transition probabilities are as follows:

$B_0^S(t) = q_{01}(t)©B_1^S(t) + \; q_{02}(t)©B_2^S(t)$

$B_1^S(t) = q_{10}(t)©B_0^S(t)$

$B_2^S(t) = W_2^S(t) + q_{20}(t)©B_0^S(t) + q_{22.4}(t)©B_2^S(t) + q_{23}(t)©B_3^S(t)$
$B_3^S(t) = q_{32}(t)©B_2^S(t)$ (11)
where $W_2^S(t) = e^{-(a\lambda_1+b\lambda_2)t}\,\overline{F(t)} + \left(a\lambda_1 e^{-(a\lambda_1+b\lambda_2)t}©1\right)\overline{F(t)} + \left(b\lambda_2 e^{-(a\lambda_1+b\lambda_2)t}©1\right)\overline{F(t)}$

Taking LT of expressions (10) & (11), solving for $B_0^{H^*}(t)$ *and* $B_0^{S^*}(t)$. The time for which server is busy due to repair and up-gradations respectively are given by

$B_0^H(t) = \lim_{s\to 0} s\,B_0^{H^*}(t) = \frac{N_3^H}{D_2}$ (12)

$B_0^S(t) = \lim_{s\to 0} s\,B_0^{S^*}(t) = \frac{N_3^S}{D_2}$ (13)

Where $N_3^H = p_{20}p_{01}W_1^{H^*}(0)$ $N_3^S = p_{02}W_2^{S^*}(0)$ *and* D_2 *is already mentioned.* (14)

3.5 Expected Number of Hardware Repairs

The expressions for $NHR_i\ (t)$ in terms of $Q_{ij}(t)$ are given as:

$NHR_0(t) = Q_{01}(t)\,Ⓢ\,[1 + NHR_1(t)] + Q_{02}(t)\,Ⓢ\,NHR_2(t)$

$NHR_1(t) = Q_{10}(t)\,Ⓢ\,NHR_0(t)$

$NHR_2(t) = Q_{20}(t)\,Ⓢ\,NHR_0(t) + Q_{22.4}(t)\,Ⓢ\,NHR_2(t) + Q_{23}(t)\,Ⓢ\,NHR_3(t)$

$NHR_3(t) = Q_{32}(t)\,Ⓢ\,NHR_2(t)$ (15)

Taking LST of equations (15) and solving for $NHR_0^{**}(s)$. The expected number of hardware repair is given by

$NHR_0 = \lim_{s\to 0} sNHR_0^{**}(s) = \frac{N_4}{D_2}$ (16)

Where $N_4 = p_{20}p_{01} + p_{02}p_{23}$ *and* D_2 *is already mentioned.* (17)

3.6 Expected Number of Software Up-gradations

The expressions for $NSU_i(t)$ in terms of $Q_{ij}(t)$ are given as:

$NSU_0(t) = Q_{01}(t)\,Ⓢ\,NSU_1(t) + Q_{02}(t)\,Ⓢ\,[1 + NSU_2(t)]$

$NSU_1(t) = Q_{10}(t)\,Ⓢ\,NSU_0(t)$

$NSU_2(t) = Q_{20}(t)\,Ⓢ\,NSU_0(t) + Q_{22.4}(t)\,Ⓢ\,NSU_2(t) + Q_{23}(t)\,Ⓢ\,NSU_3(t)$

$NSU_3(t) = Q_{32}(t)\,Ⓢ\,NSU_2(t)$ (18)

Taking LST of expressions (18) and solving for $NSU_0^{**}(s)$. The expected numbers of software up-gradation are given by

$NSU_0(\infty) = \lim_{s\to 0} sNSU_0^{**}(s) = \frac{N_5}{D_2}$ (19)

Where $N_5 = p_{02}p_{20}$ *and* D_2 *is already mentioned* (20)

4. Profit Analysis

In steady state, the profit earned to the system model can be obtained by the formula:

$P = K_0A_0 - K_1B_0^H - K_2B_0^S - K_3NHR_0 - K_4NSU_0$ (21)

Where $K_0 = 15000, K_1 = 1000, K_2 = 700, K_3 = 1500, K_4 = 1200$

5. Particular Cases

Suppose $g(t) = \alpha e^{-\alpha t}$ *and* $f(t) = \theta e^{-\theta t}$

$N_1 = \frac{a\lambda_1+2b\lambda_2+\theta}{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\theta)}$ $D_1 = \frac{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\theta)-\theta b\lambda_2}{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\theta)}$

$N_2 = \frac{b\lambda_2+\theta}{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\theta)}$ $D_2 = \frac{\theta^2(a\lambda_1+\alpha)+\alpha(a\lambda_1+b\lambda_2+\theta)b\lambda_2}{\alpha\theta(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\theta)}$

$$N_3^H = \frac{\theta a\lambda_1}{(a\lambda_1+b\lambda_2)\alpha(a\lambda_1+b\lambda_2+\theta)} \qquad N_3^S = \frac{b\lambda_2}{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\theta)}$$

$$N_4 = \frac{a\lambda_1(a\lambda_1+\theta)}{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\theta)} \qquad N_5 = \frac{b\lambda_2(a\lambda_1+\theta)}{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\theta)}$$

6. Conclusion

The behaviour of some important reliability measures of the system model such as mean time to system failure (MTSF), availability and profit has been shown numerically respectively in tables 1 to 3. It is found that these measures keep on decreasing with the increase of failure rates (λ_1 and λ_2) while their values increase with the increase of hardware repair rate (α), software up-gradation rate (θ) and by interchanging the values of a & b (a < b).

7. Numerical Presentation of Reliability Measures

Table 1: MTSF Vs Hardware Failure Rate (λ_1)

λ_1	λ_2 =0.001,α=2,θ=5, a=0.6,b=0.4	λ_2=0.002	θ=7	a=0.4,b=0.6
0.01	166.665779	166.6631166	166.6660324	249.9955048
0.02	83.33311168	83.33244687	83.33317489	124.9988771
0.03	55.55545716	55.55516204	55.5554852	83.33283465
0.04	41.66661139	41.66644558	41.66662712	62.49971972
0.05	33.333298	33.333192	33.33330805	49.99982076
0.06	27.77775327	27.77767975	27.77776023	41.66654229
0.07	23.80950582	23.80945187	23.80951093	35.71419441
0.08	20.83331958	20.83327832	20.83332348	31.24993015
0.09	18.51850766	18.51847511	18.51851074	27.77772263
0.1	16.66665788	16.66663154	16.66666037	24.99995537

Table 2: Availability Vs Hardware Failure Rate (λ_1)

λ_1	λ_2=0.001,α=2,θ=5, a=0.6,b=0.4	λ_2=0.002	α=3	θ=7	a=0.4,b=0.6
0.01	0.99700911	0.997009	0.998004	0.997009	0.99800412
0.02	0.994036064	0.994036	0.996016	0.994036	0.99601621
0.03	0.991080696	0.991081	0.994036	0.991081	0.9940362
0.04	0.988142849	0.988143	0.992064	0.988143	0.99206404
0.05	0.985222367	0.985223	0.990099	0.985222	0.9900997
0.06	0.982319099	0.98232	0.988143	0.982319	0.98814312
0.07	0.97943289	0.979434	0.986194	0.979433	0.98619426
0.08	0.976563592	0.976565	0.984252	0.976563	0.98425307
0.09	0.973711057	0.973712	0.982319	0.973711	0.98231951
0.1	0.970875138	0.970876	0.980393	0.970875	0.98039353

Table 3: Profit (P) Vs Hardware Failure Rate (λ_1)

λ_1	λ_2=0.001, α=2, θ=5, a=0.6, b=0.4	λ_2=0.002	α=3	θ=7	a=0.4, b=0.6
0.01	14945.59	14945.06	14960.51	14945.61	14963.25317
0.02	14891.97	14891.43	14921.63	14891.98	14927.42194
0.03	14838.59	14838.06	14882.84	14838.61	14891.70168
0.04	14785.47	14784.94	14844.14	14785.48	14856.09188
0.05	14732.58	14732.06	14805.52	14732.6	14820.59203
0.06	14679.95	14679.42	14766.98	14679.95	14785.20163
0.07	14627.55	14627.02	14728.53	14627.55	14749.92017

0.08	14575.39	14574.87	14690.17	14575.39	14714.74715
0.09	14523.47	14522.95	14651.89	14523.47	14679.68206
0.1	14471.79	14471.27	14613.7	14471.78	14644.72442

8. Comparative Study of Profits of the System Models

The profit of the present model has been compared with that of the model Munday and Malik (2015). It is observed that the present model is less profitable as compared to that model. Thus, in a computer system with software redundancy in cold standby, the idea of priority to hardware repair over software up-gradation is not helpful in increasing the profit of the system if system has more chances of hardware failure than that of software failure (a>b). However, this idea can be helpful in improving the profit of a computer system which has less chances of hardware failure as compared to software failure. And, in that situation the profit difference of the system models goes on increasing with the increase of hardware failure rate. The profit difference (P-P1) of the models is shown numerically in table 4.

9. Numerical Presentation of Profit Difference (P – P1)

Table 4: (P–P1) Vs Hardware Failure Rate (λ_1)

λ_1	λ_2=0.001, α=2, θ=5, a=0.6, b=0.4	λ_2=0.002	α=3	θ=7	a=0.4, b=0.6
0.01	-15.6815831	-15.4128745	-11.39364555	-15.69047607	20.61784182
0.02	-31.53207673	-31.26288897	-23.04367167	-31.54195767	41.27628135
0.03	-47.28423293	-47.01456637	-34.68146501	-47.2953841	61.70976563
0.04	-62.93887394	-62.66872919	-46.30650628	-62.95171252	81.92077038
0.05	-78.49681575	-78.22619364	-57.91829329	-78.51188892	101.9117387
0.06	-93.95886812	-93.68776969	-69.51634047	-93.97684822	121.6850814
0.07	-109.3258346	-109.0542611	-81.10017841	-109.3475145	141.2431778
0.08	-124.5985124	-124.3264653	-92.66935339	-124.624801	160.5883759
0.09	-139.7776927	-139.5051736	-104.2234269	-139.8096104	179.7229931
0.1	-154.8641603	-154.5911711	-115.7619754	-154.9028351	198.6493166

10. References

[1] Welke, S. R.; Labib, S. W.; Ahmed, A. M. (1995): Reliability modeling of hardware/software system, IEEE Transactions on Reliability, 44(3), pp.413-418.

[2] Yadavalli, V.S.S.; Chanderasekhar; Natarajan, R. (2004): A study on two-unit standby system with Erlangian repair time, Asia-Pacific Journal of Operational Research, 21(3), pp. 271-277.

[3] Anand, Jyoti; Malik, S.C. (2012): Analysis of a computer system with arbitrary distributions for h/w and s/w replacement time and priority to repair activities of h/w over replacement of the s/w, International Journal of Systems Assurance Engineering and Management, 3(3), pp. 230-236.

[4] Kumar, Jitender; Kadyan, Mukender Singh; Malik, Suresh Chander (2012): Cost-analysis of a two-unit cold standby system subject to degradation, inspection and priority, Maintenance and Reliability, 14(4), pp. 278-283.

[5] Barak, Sudesh K.; Malik, S.C. (2014): A standby system with priority to repair over preventive maintenance, International Journal of Engineering Research (IJER), 3(4), pp. 241-244.

[6] Kumar, Ashish; Barak, Monika S; Malik, S.C. (2015): Economic analysis of a computer system with software up-gradation and priority to hardware repair over hardware replacement subject to maximum operation and repair times, International Journal of Scientific and Engineering Research, 6(4), pp. 1661-1668.

[7] Munday, V.J.; Malik, S.C. (2015): Stochastic modelling of a computer system with software redundancy, International Journal of Engineering and Management Research, 5(1), pp.295-302.