MODIFIED ORDERING POLICY OF AN EOQ MODEL WITH ALLOWABLE PROPORTIONATE DISCOUNT FOR IMPERFECT PRODUCTS USING CROSS SELLING EFFECTS AND DATAMINING TECHNIQUES

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Abstract

The goal of this work is to provide an Economic Order Quantity (EOQ) model with allowable proportionate discount of imperfect quality items and cross selling effects. We first developed the ordering policy in this study by introducing cross-selling impact. Under the cross-selling effects, the frequent item sets are considered as individual entities and their corresponding EOQ can be computed further with. Moreover, when items are defective in nature the cross-selling effects remain more prominent. To establish the desired relationship among the item sets, different data mining techniques are explored initially. The work progresses further with the application of the cross-selling effects to estimate the EOQ. The work conducts a 100% screening of lot, as each lot assumed to have a fraction of imperfection. Then after variable discounts allowed for the imperfect items to find total profit. Finally, numerical analysis with a few examples is given to experience the outcomes of presented model.

Keywords: **Frequent item sets; imperfect quality items; cross selling effects; proportionate discount.**

1. Introduction

Procurement, availability, and processing of data to fetch the desired information has been an emerging area of research due to increasing demand in many vivid application domains that includes industries, social sectors and business establishments. Use of a suitable data mining technique helps these organizations for their expansion, growth and strategic decision-making attributes. The prime objective of data mining is to process a large chunk of raw data so as to fetch the desired and relevant information. Business entities can use the previously unknown relationship among the data to develop new advertising strategies so as to forecast effective selling or marketing of a product. The growth and utilization of modern computer resources in the area of marketing has created the requisite relationship between the customer and management. It helps to gain competitive advantage in firms or profitable organizations. The accessibility to such resources provides innovative ideas for data mining. Subsequent exploration results an efficient management of data mining tools by enhancing traditional methods. Implementation of such new data mining approaches is essential for effective production planning and inventory control that motivates the authors to move in this direction.

The technique of obtaining usable information from enormous amounts of data stored in databases is known as data mining. A most significant aspects of data mining is Association rule mining, that discovers meaningful associations among a large number of data objects while keeping track of business transaction details. Again, Clustering is grouping of a number of transactions into clusters with identical properties within the same cluster and unlike transactions are in distinct clusters. When we enter into inventory of huge size, it's become a challenging task to find EOQ of each item. Hence, it will become easier when items of inventory are categorised into different groups which we can say as Classification. Association rule mining, clustering, classification techniques help in effective inventory management while modelling EOQ of inventory. Practically, as there exist inter relation among certain items, sales of one affect another. Depending upon strength of their relationship, there may cause loss of sales which is known as cross selling effect. The lost cost because of this effect is termed as opportunity cost.

In today's competitive business world, in spite of well-planned manufacturing, control, and highly developed production methods; the fraction of the produced items may contain some defects. The imperfect quality items may not be defective always and can be used in another inventory situation. One of the best examples is electronics industry. Generally, it is found that the imperfect quality items have got direct effect on the inventory management. So, a 100% screening is necessary for the defective items before these reach the customers. Different researchers have presented their ideas for modelling EOQ of imperfect quality items in several ways to address such inventory situations.

The remainder of this paper is organised as follows: Section 2 is gives the review of literatures studied. Section 3 provides a brief explanation of the work done. Section 4 represents Mathematical formulation of the proposed model with data mining techniques as, apriori, clustering, ABC Classification. Section 5 provides numerical analysis with different examples. Section 6 contains conclusion based on our model.

2. Literature Review

Porteus (1986) presented the first fundamental EOQ model in order to estimate the impact of defectives. Jaber et al. (2000) used a classic EOQ strategy of selling defective items at a fixed rate of discount as one batch after 100 percent screening was completed. Jaggi et al. (2013); Jaggi and Mittal (2012) have further updated the traditional EOQ approach for the benefit of industries and organizations during the last two decades. A study on these researches supports an adequate classification of inventories in order to decide different inventory policies to demarcate the items to their designated classes. Anand et al. (1997) presented ideas for some inventory classes, an item is not essentially categorized into a designated class and is influenced by other similar or complementary items. Such an effect is popularly designated as the "cross-selling effect" which changes the EOQ of an inventory item forcefully. This has led to the development of several association rules mining algorithms in this direction Agrawal et al. (1993). Further a new idea based on clustering of transactions with homogeneous items rather using any pair-wise similarity given by Wang et al. (1999). A few worth mentioning EOQ inventory model developments considering the cross-selling effects have been further made by Kaku (2004); Kaku and Xiao (2008) for better inventory management and control. In this regard, Bala (2009) proposed a multi-item inventory model for retail selling with efficient result. A work has been proposed in order to compare different association rule mining techniques by Khurana and Sharma (2013). Similarly, the authors in Mittal et al. (2014) built an inventory model to compute EOQ for imperfect products with cross-selling impacts using association rule mining. Further Mittal et al. (2015) focused on developing an enhanced model for imperfect inventory items in consideration with impact of cross-selling, clustering and association rule mining. In modelling of EOQ a new classification technique ABC and cross selling effect introduced that modifies the inventory policy for multi-item inventory by Mittal et al. (2017). Use of the learning effect in establishing an economic inventory model that provides a proportionate discount for items of defective quality has laid new foundation in this direction Patro et al. (2017). Agrawal et al. (2018) presented an inventory model for a retailer's ordering policy with different thresholds at different levels along with multi-level association rule mining to obtain frequent item-sets at each level. Singh et al. (2020) studied a model where the opportunity cost of an item is changed by accounting for purchase dependency. Partial backordering is also permitted in this model. Singh et al. (2022) further worked to modify EOQ model in retail multi-item inventory management using purchasing dependencies. They used the amount of loss in profit to change an item's opportunity cost.

The survey of literature shows the application of cross-selling effects in establishing EOQ models with constant discount for percentage of defectives, with 100% screening. It aims to find the percentage of screening present in each lot of the EOQ inventory model to determine the profit incurred with due attention to an optimal order or lot size. Nevertheless, the absence of a proportionate discount in defective items percentage attributes to the cross-selling effects with various data mining techniques in modeling the inventory creates new research direction for this work.

3. Proposed Work

This paper conceptualizes the EOQ inventory model for development of imperfect items differently by taking impact of cross-selling by means of three different cases such as few association rules, association rule mining along with clustering and ABC classification. Moreover, variable discount is used as each individual sample assumed to have a certain % of imperfections. For finding the total profit, the work conducts a 100% screening of the lot under consideration with an allowable proportionate discount for the imperfect items. It aims to resolve the real-world business problems by comparing modified inventory policies in line with the cross-selling effects in a Support-Confidence framework between each individual case considered and existing policies. A new estimation approach is further proposed to include the opportunity cost in the baseline of the association rules to compute the inventory models of imperfect items in a set of frequently used items. Subsequently, it explores a new opportunity cost so as to modify the frequent item inventory model as a multi-item set. To validate the proposed approach, the work provides a numerical example for illustration.

4. Model of Proposed Work

This work investigates the cross-selling impacts in conjunction with various data mining approaches and provides a difference among the order amounts for defectives by applying variable rate of discount in frequent itemset.

In situations where there exists a close relationship among different products, the sale of one is dependent on the sale of other. Consider the provided item set f, which includes the items $\{r1, r2, r3, \ldots rn\}$. In this the "Support" of an item means it's frequency of occurrence in the whole transactions. For item r1 it can be expressed as:

$$\text{Support}(r1) = \text{Frequency}(r1) / \text{Total number of transactions} \quad (1)$$

The relationships between items can be specified by "Confidence" or the conditional probability.

conf ($r1 \rightarrow r2$) refers to frequency at which $r2$ is purchased while $r1$ is purchased.

$$\text{Conf}(r1 \rightarrow r2) = \text{Support}(r1 \text{ U } r2)/\text{Support}(r1) \quad (2)$$

With apriori algorithm it can be established that a frequent itemset based on support confidence framework. It indicates the association rules for the items generated with higher support and confidence than user-defined minimum support and confidence. In Apriori, algorithm, items for frequent itemset can be find out on the basis of minimum support as well as the generate association rules on basis of the threshold confidence.

The algorithm depicted in this direction is worth noting and can be materialized using the following stages Agrawal et al. (1993).

Stage 1: This step checks entire transactions to count the number of times each item appears in order to identify a set of frequently occurring items.

Stage 2: this step constitutes of two stages: apriori generation and support of candidate counting. First, L_{k-1} is used to generate candidate itemset C_k in order to get L_k, after which the database is examined and candidate support counts are compared with minimum support count by checking the condition, $k \geq 2$. Join and prune are two activities that the apriori generation function is utilized for. Join step joins Lk-1 with Lk-1 for generating prospective candidates. However, the prune phase employs the Apriori property to exclude items with infrequent subsets.

We also considered another case as clustering of transactions Wang et al. (1999) for the analysis of present work. Here, the term "large items" holds items from a few numbers of transactions in a cluster under similarity basis. In cluster Ci, the support of an item is specified by total transactions in Ci. Thus, in a cluster large item presents are homogeneous in nature and also support is at least equal to s × Ci, where s is user defined minimum support otherwise small which are heterogeneous. Aim of considering this clustering is to minimize the cost. Moreover, minimized cost C is computed by using two types of cost factors as:1st one intra-cluster cost which is calculated by all small items, 2nd one inter-cluster cost i.e., alias of large items in all clusters. In this process of clustering, dynamically clusters are created and eliminated based on cost optimization.

This clustering algorithm Wang et al. (1999) specified in terms of two phases as:

(1) Allocation phase: sequentially every transaction is read and assigned to a cluster. Either the cluster is existing one or new.
(2) Refinement phase: minimizes the cost

In the third case of this work, a classification approach namely ABC classification is considered for modifying EOQ inventory policies for multi-item. Aim is to analyze the effect of cross-selling factor on items of individual groups. On basis of Pareto principle which states that a tiny proportion of items transaction for higher % of total dollar usage (product of unit price and annual demand of item), this technique first classifies the list of items of a given inventory database into three groups as A, B and C. A refers to significant few and C refers to trivial many.

Then after cross selling effect is obtained with the help of a well-known scheme of data mining i.e., association rule mining in each of three categories.

The effect of cross on items is represented by the corresponding confidence between items. In frequent itemset, the effect of an out-of-stock item rk on another item i i.e, $f(r1, r2, r3 \ldots..rn)$ can be represented as a probability

$$\Pr ob_{k,i} = \sum_{i=1}^{n} conf\left(k \to f(k,i)\right) \tag{3}$$

where $k = 1,2,3,......n$ denotes the items present in a frequent item-set and $f(k,i)$ represents the subset of item i not including k number of items. For $i = k$, $f_{i,i} = i$ that gives $conf(i \to i) = 1$.

The opportunity cost of an item k can be described by the lost cost of that item on account of the effect of cross selling. It can be represented by the relation

$$OC_k = \sum u_i . prob_{k,i} \tag{4}$$

where $u_i =$ cost of each unit item i. In this regard the Probabilistic index I_{ndk} which is defined as:

$$I_{ndk} = \frac{OC_k + H_k}{OC_k} \tag{5}$$

where H_k = cost of holding item k per unit. The I_{ndk} will be used later part to modify order policy along with opportunity cost.

The present work analyzed mathematically in modelling of EOQ with imperfect items by considering some assumptions which are closer to realistic situations.

Now, Consider the products in the frequent item set that are supplied instantly with an order size of Z.

The number of perfect items are given as,

$$Z - pZ = (1-p)Z \tag{6}$$

To overcome lack of perfect items $(1-p)Z$ at the time of screening t must be at least equal to the demand. Jaber et al., [2]. i.e.,

$$(1-p)z \geq Dt, \tag{7}$$

$$p \leq 1 - \frac{D}{w} \tag{8}$$

The total cost per cycle is:

$$TC(Z) = C_k + C_p Z + C_s Z + C_h \times \left(\frac{Z(1-p)T}{2} + \frac{pZ^2}{w} \right) \tag{9}$$

Total sales of perfect and imperfect items are added to calculate the total revenue per cycle i.e., given by,

$$TR(Z) = \frac{2S_g Z^2 + C_k + C_p Z + C_s Z + C_h \times \left(\frac{Z(1-p)T}{2} + \frac{pZ^2}{w} \right)(Zp+1)}{2Z + (Zp+1)} \tag{10}$$

Per cycle, total profit is =

$$TP(Z) = TR(Z) - TC(Z) \tag{11}$$

$$TP(Z) = \left\{ \frac{2S_g Z^2 + \left\{ C_k + C_p Z + C_s Z + C_h \times \left(\frac{Z(1-p)T}{2} + \frac{pZ^2}{w} \right) \right\}(Zp+1)}{2Z + (Zp+1)} \right\} - \left[C_k + C_p Z + C_s Z + C_h \times \left(\frac{Z(1-p)T}{2} + \frac{pZ^2}{w} \right) \right] \tag{12}$$

Per unit, total profit $TPU(Z)$ is computed by

$$TPU(Z) = TP(Z)/T \tag{13}$$

Where $T = \frac{Z(1-p)}{D}$

$$TPU(Z)=\frac{2D\left(S_gZ-C_k-C_pZ-C_sZ\right)}{2Z+Zp+1}\left(\frac{1}{1-p}\right)-\frac{C_hZ^2}{2Z+Zp+1}(1+p) \tag{14}$$

As p is random with a known probability density function $f(p)$, $ETPU(Z)$ can be represented as:

$$ETPU(Z)=\frac{2D\left(S_gZ-C_k-C_pZ-C_sZ\right)}{2Z+ZE[p]+1}E\left(\frac{1}{1-p}\right)-\frac{C_hZ^2}{2Z+ZE[p]+1}\left(1+E[p]\right)$$
$$-\frac{C_hZ^2}{2Z+ZE[p]+1}\left(1+E[p]\right) \tag{15}$$

The optimality requirement in this case denotes the concavity of $ETPU(Z)$, which is calculated with 1st derivative of Eq. (15) i.e.,

$$ETPU'(Z)=\left(\frac{1}{\left(2Z+ZE[p]+1\right)^2}\right)\begin{bmatrix}2DS_g-2DC_p-2DC_s+4DC_k\\+2DC_kE[p]E\left(\dfrac{1}{1-p}\right)-2C_hZ^2\\-C_hZ^2\left(E[p]\right)^2-3C_hZ^2E[p]\\-2C_hZ-2C_hZE[p]\end{bmatrix} \tag{16}$$

The second derivative of Eq. (15)

$$ETPU''(Z)=-\left(\frac{2}{\left(2Z+ZE[p]+1\right)^3}\right)\left(\left(2+E[p]\right)\begin{pmatrix}2DS_g-2DC_p-2DC_s+4DC_k\\+2DC_kE[p]\end{pmatrix}E\left(\frac{1}{1-p}\right)+C_h+C_hE[p]\right) \tag{17}$$

The 2nd derivative of $ETPU(Z)$ gives -ve for all values of z, that implies existence of a distinct value of Z_{max} that maximizes Eq. (13) which is given as follows.

$$Z_{\max}=\sqrt{\frac{(2DS_g-2DC_p-2DC_s+4DC_k+2DC_kE[p]E\left(\dfrac{1}{1-p}\right)}{2C_h+3C_hE[p]+C_h(E[p])^2+\dfrac{2C_h}{Z}+\dfrac{2C_h}{Z}(E[p])}} \tag{18}$$

For large value of Z, $\frac{1}{Z}\rightarrow 0$

$$Z_{\max}=\sqrt{\frac{(2DS_g-2DC_p-2DC_s+4DC_k+2DC_kE[p]E\left(\dfrac{1}{1-p}\right)}{2C_h+3C_hE[p]+C_h(E[p])^2}} \tag{19}$$

Z_{max} provides the order quantity for item set. To find an optimal order quantity, this work modifies the order quantity by taking the effects of cross selling. In order to find the modified order quantity for an imperfect frequent itemset, Eq. (5) is modified as follows:

$$EOQ=Z_{\max}\sqrt{I_{nd}} \tag{20}$$

5. Numerical Example

A variety of factors are taken examined in order to work out the proposed task, including cost, selling price, demand each year, and so on.

We considered 3 different numerical examples to validate the present work. The data taken from Mittal et al. (2014), Mittal et al. (2015), Mittal et al. (2017) and Patro et al. (2017) for Apriori, association rule mining with clustering and classification. We have made a comparative analysis among the methods mentioned. We have executed the algorithms in the MATLAB platform and the equations have been analyzed using Mathematica 5.1.

In each case an assumption taken that the defective p is uniformly distributed with probability density function. The expected values are given as follows:

$E[p]=0.02$ and $E\left[1/(1-p)\right]=1.02055$

5.1. Case I : Apriori

To illustrate the developed model, we adopted the values of the parameters as shown in Table 1 according to Mittal et al. (2014) and analyze the inventory situation:

Minimum support or min_sup	50%
Minimum confidence or min_conf	60%
C_k	100 per cycle
C_h	\$10 per unit per year
C_s	\$1 per unit
S_g	\$60 per unit
w	131400 units/year

Table 1. Set Parameters of Inventory Situation

Suppose the inventory item-set be $I = \{r_1, r_2, r_3, r_4, r_5, r_6\}$, then the inventory transaction set can be given by TID={1500, 2500, 3500, 4500, 5500} as shown in Table 2. The inventory transactions are provided in the rows of this Table.

TID	ITEMS
1500	r_1 r_3 r_4
2500	r_2 r_3 r_5
3500	r_1 r_2 r_3 r_5
4500	r_2 r_5
5500	r_4 r_6

Table 2. An Inventory Transaction Data Base

The Support-Confidence framework can be used to identify the association rule of these inventory transactions. The apriori approach is used to find the most frequently occurring item sets in the transaction data base. as follows:

$$\{r1\},\ \{r2\},\ \{r3\},\ \{r4\},\ \{r5\},\ \{r1\ r3\},\ \{r2\ r3\},\ \{r2\ r5\},\ \{r3\ r5\},\ \{r2\ r3\ r5\}$$

In Table 3 the inventory policy in frequent item-set $\{r_2, r_3,, r_5,\}$ is considered. The most frequent items are given as $\{r_2, r_3,, r_5,\}$ with a support which is greater than the min_sup. It is chosen form various classes of items. According to the apriori algorithm a min_sup of 50% is considered. By taking the min_conf of 60%, the confidence of items and their subsets in $\{r_2, r_3,, r_5,\}$ remains larger than that. In the same way, the confidence of the other frequently used item sets are calculated and specified in Table 4.

Item	D	C_p
r_2	50,000	30.00
r_3	40,000	20.50
r_5	45000	45.52

Table 3. Parameter values in Inventory Policy (Apriori)

ITEMS	CONFIDENCE
$r_2 \to r_3$	66.7%
$r_2 \to r_5$	100%
$r_2 \to r_3 \cup r_5$	66.7%
$r_3 \to r_2$	66.7%
$r_3 \to r_5$	66.7%
$r_3 \to r_2 \cup r_5$	66.7%
$r_5 \to r_2$	100%
$r_5 \to r_3$	66.7%
$r_5 \to r_2 \cup r_3$	66.7%

Table 4. Rules with Confidence

To calculate the opportunity cost of the frequent itemset $\{r_2, r_3,, r_5,\}$, the following formulae have been used which is given in Eq. (4).

$$OC_{r2} = C_{r2}.conf(r_2 \to r_2) + C_{r3}.\{conf(r_2 \to r_3) + conf(r_2 \to r_3 \cup r_5)\} + C_{r5}.\{conf(r_2 \to r_5) + conf(r_2 \to r_3 \cup r_5)\}$$
$$= 30 \times 1 + 20.5 \times \{0.667 + 0.667\} + 45.52 \times \{1 + 0.667\}$$
$$= 133.22884$$

Similarly,

$OC_{r3} = 121.24368$ and $OC_{r5} = 122.877$

The order policy for items r2, r3, r5 has been modified by substituting values of opportunity cost in Eq. (5) as,

$$I_{nd_{r2}} = \frac{OC_{r2} + H_r}{OC_{r2}} = 1.075058824$$

Similarly,

$I_{nd_{r3}} = 1.082478526$ and $I_{nd_{r5}} = 1.081382195$

Now the optimal value of Z_{max}, $ETPU(Z)$ for item r_2 are computed and given as follows:

$Z_{max} = 1069.66$ and $ETPU(Z) = 1454344.943$

Therefore, EOQ of item r_2 modified as:

$EOQ = Z_{max}\sqrt{I_{nd}}$ =1109.077436

In the similar way we also calculated for item r_3, r_5 and are given in Table 5. These values are compared with the existing models. Then represented in Table 4 and Fig.1

Items	Z_{max}	$ETPU(Z)$	EOQ
r_2	1069.66	1454344.943	1109.077436
r_3	976.211	1546227.476	1015.452148
r_5	980.09	603043.1309	1019.190966

Table 5. Modified value with Apriori

Items	**Traditional EOQ**	**EOQ (Mittal et al.)**	**EOQ (Present work)**
r_2	1000	1049.88991	1109.077436
r_3	899.427191	943.6349189	1015.452148
r_5	948.6832981	1000.575114	1019.190966

Table 6. Comparison with existing models

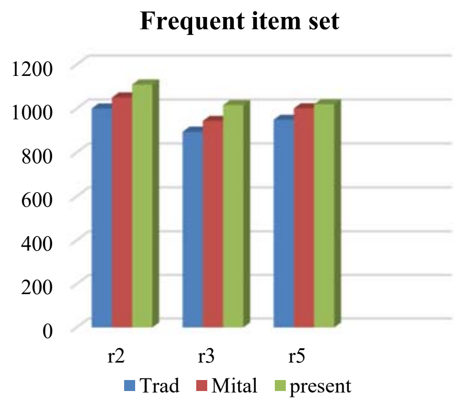

Fig. 1. Comparison of EOQ values obtained in proposed method (Apriori) with the state-of-art methods

5.2. *Case-II: association rule mining with clustering*

In this case first homogeneous clusters are obtained using Clustering algorithm on an inventory database. Then after apriori algorithm is applied on clustered data for generating rules of association. Further, opportunity cost is formulated to compute EOQ for the products having imperfectness. To analyse the inventory policy presented in this work we used the following parameter values.

min_sup=60%, min_conf=75%, O_c =100/cycle, H_c = \$5/unit/year, r =1unit/min, S_c =0.5/unit, P_c =25/unit, B_c = \$ 20/unit, S = \$50/unit, w = 175 200 units/year.

Let us take a database set D_s and the inventory item-set, $I_s = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$. Each row indicates an inventory transaction in the set, ITD = {ITD 1, ITD 2, ITD 3, ITD 4, ITD 5, ITD6}, that is given in Table 7.

ITD	ITEMS
ITD1	x_1 x_2 x_3
ITD2	x_1 x_2 x_3 x_4
ITD3	x_1 x_2 x_3 x_5
ITD4	x_1 x_2 x_6
ITD5	x_4 x_7 x_8
ITD6	x_4 x_7 x_9

Table 7. An Inventory Transaction Database

The parameter values taken for inventory policy to compute the opportunity cost of frequent items are represented in Table 8.

Item	D	C_p	C_h	S_g	S_{imp}
x_1	50,000	30.00	3	60.00	25.00
x_2	40,000	20.50	2	40.00	17.00
x_3	40,000	45.52	4	60.00	40.00
x_4	50,000	50.00	5	90.00	45.00
x_5	50,000	45.52	4	70.00	40.00
x_6	40,000	40.00	4	65.00	35.00
x_7	50,000	34.00	3	60.00	30.00
x_8	40,000	32.00	3	50.00	27.00
x_9	50,000	23.00	2	40.00	18.00

Table 8. Parameter values for inventory policy

In the inventory transaction database, an assumption taken as minimum support is 60%, total number of transactions = 6. Now here a large item is obtained by the formula (i.e., 6 *60%) and have at least 4 transactions. Similarly, we have calculated for all clusters and given in Table 9.

Clustering	**Large**	**Small**
C1 ={itd1,itd2, itd3,itd4, itd5, itd6}}	L1= $\{x_1, x_2\}$	S1= $\{x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$
C2= {C1={itd1,itd2,itd3,itd4}, C2={itd5,itd6}}	L1= $\{x_1, x_2, x_3\}$ L2= $\{x_4, x_7\}$	S1= $\{x_4, x_5, x_6\}$ S2= $\{x_8, x_9\}$
C3= {C1 = {itd1, itd2}, C2 = {itd3, itd4}, C3 = {itd5,itd6}}	L1= $\{x_1, x_2, x_3\}$ L2= $\{x_1, x_2\}$ L3= $\{x_4, x_7\}$	S1= $\{x_4\}$ S2= $\{x_3, x_5, x_6\}$ S3= $\{x_8, x_9\}$

Table 9. Clustering details

Therefore, cluster C2 is considered as cost obtained is minimum in comparison to cluster C1 and C3. So, the transactions in the given database are grouped into two clusters as, C1 = {itd1, itd2, itd3, itd4} and C2 = {itd5, itd6}. Then apriori algorithm is applied on both clusters to obtain frequent sets. $\{x_1, x_2, x_3\}$ and $\{x_4, x_7\}$ are the most frequent item-set in cluster C1 and C2 respectively. Confidence of items in both frequent item-set are calculated with Eq. (2) and given in Table 10.

C_1		C_2	
Items	**Confidence**	**Items**	**Confidence**
$x_1 \rightarrow x_2$	100	$x_4 \rightarrow x_7$	100
$x_1 \rightarrow x_3$	75	$x_7 \rightarrow x_4$	100

$x_1 \to x_2 \cup x_3$	75	
$x_2 \to x_1$	100	
$x_2 \to x_3$	75	
$x_2 \to x_3 \cup x_4$	75	
$x_3 \to x_1$	100	
$x_3 \to x_2$	100	
$x_3 \to x_1 \cup x_2$	100	

Table 10. Confidence of frequent items in cluster C_1 and C_2

Then the opportunity cost in the frequent itemset $I = \{x_1, x_2, x_3, x_4, x_7\}$ can be calculated by formulae given in Eq. (4).

Opportunity cost of item $x_1 =$

$$
\begin{aligned}
OC_{(x_1)} &= C_{x_1}.conf\left(x_1 \to x_1\right) + C_{x_2}.\left\{conf\left(x_1 \to x_2\right) + conf\left(x_1 \to x_2 \cup x_3\right)\right\} \\
&\quad + C_{x_3}.\left\{conf\left(x_1 \to x_3\right) + conf\left(x_1 \to x_2 \cup x_3\right)\right\} \\
&= 30 \times 1 + 20.50 \times \{1 + 0.75\} + 45.52 \times \{0.75 + 0.75\} \\
&= 30 + 35.875 + 68.28 \\
&= 134.155
\end{aligned}
$$

Now, I_{nd} (index) of item x_1 formulated using this opportunity cost as,

$$I_{nd(x_1)} = \frac{H_r + OC_{(x_1)}}{OC_{(x_1)}} = 1.07534221$$

Similarly, for the frequent items of both Clusters C_1 and C_2, opportunity cost and index value are calculated. Then after the optimal value of $Z_{\max}$, $ETPU(Z)$, and EOQ of items of frequent item sets for both clusters can be modified as given by Table 11. Comparison of these values with the existing models are represented in Table 12. and Fig. 2.

Items	OC_k	I_{nd}	$Z_{\max}$	$ETPU(Z)$	EOQ
x_1	134.155	1.07534221	1952.93	1459235.363	2025.163184
x_2	141.28	1.070781427	2090.14	743508.6028	2162.846964
x_3	146.52	1.06825007	1461.03	538932.4641	1510.064851
x_4	84	1.11904762	1545.13	1962567.359	1634.516498
x_7	84	1.11904762	1935.95	1257192.183	2047.945619

Table 11. Modified values with association rule mining clustering

Items	**Traditional EOQ**	**EOQ (Mittal et al.)**	**EOQ (Present work)**
x_1	1825.741858	1915	2025.163184
x_2	2000	2099	2162.846964
x_3	1414.213562	1483	1510.064851
x_4	1414.213562	1474	1634.516498
x_7	1825.741858	1881	2047.945619

Table 12. Comparison of the values with clustering and existing models

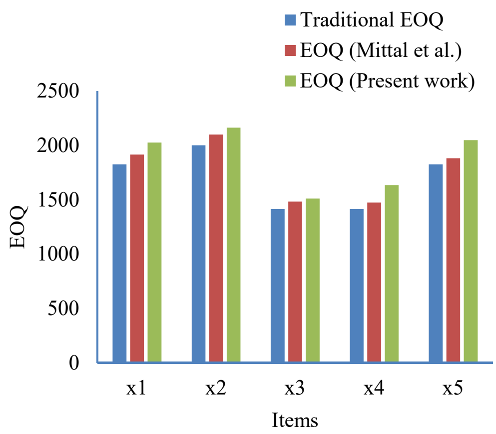

Fig. 2. Comparison of EOQ values obtained in proposed method (association rule mining with clustering) with the state-of-art methods

5.3. *Case-III: Classification*

We considered another example for analysis of the present work. ABC classification is used to categorize the inventory items into three sets namely A, B, and C depending on their dollar usage. Further, opportunity cost is formulated to compute EOQ for imperfect quality items. Following are the few parameter values that we used to analyze the presented model.

O_c=100/cycle, H_c= 25% of unit cost, r =1unit/min, S_c =5% of unit cost, w = 120000 units/year

Let us Consider the inventory database set ID that contains inventory item-set, $ID=\{t_1,t_2,t_3,t_4,t_5,t_6,t_7,t_8,t_9\}$. Each row indicates an inventory transaction in the set, TID = {T1, T2, T3, T4, T5, T6, T7, T8, T9}, that is given in Table-13. The parameter values considered for inventory policy to compute the opportunity cost of items are represented in Table 14, Table 15 represents ABC classifications for the items give in Table13 with respect to conditions.

TID	**Transactions**
IT1	t_1 t_2 t_4
IT2	t_1 t_3 t_5 t_6
IT3	t_2 t_4 t_6 t_8 t_9
IT4	t_3 t_5 t_7
IT5	t_2 t_4 t_8 t_9
IT6	t_1 t_2 t_3 t_4
IT7	t_2 t_5 t_7 t_8 t_9
IT8	t_2 t_3 t_6 t_7
IT9	t_4 t_9

Table 13. An Inventory Transaction Database

Item	C_p	D	Dollar	S_g	Sip	C_h	C_s
t_1	10	7000	70000	20	12	2.5	0.5
t_2	15	1500	22500	30	25	3.75	0.75
t_3	6	10000	60000	10	8	1.5	0.3
t_4	10	1000	10000	30	20	2.5	0.5
t_5	11	3500	38500	25	15	2.75	0.55
t_6	7	10000	70000	20	10	1.75	0.35
t_7	7	5000	35000	15	10	1.75	0.35
t_8	10	3000	30000	10	15	2.5	0.5
t_9	10	1000	10000	25	20	2.5	0.5

Table 14. Parameter values for inventory policy with dollar usage

Classification groups	**Items**
A (dollar usage >= 60000)	t_1 t_3 t_6
B (dollar usage 30000-60000)	t_5 t_7 t_8
C (dollar usage <30000)	t_2 t_4 t_9

Table 15. ABC classification of inventory items

Now to find EOQ of each class, we need to consider the transactions containing items of same class, is given in Table 14. Applying apriori algorithm support and confidence of items of each group are calculated in order to obtain the opportunity cost and specified in Table 17. For items of
group A, $sup(t_1) = 3$, $sup(t_3) = 4$, $sup(t_6) = 3$,
group B, $sup(t_5) = 3$, $sup(t_7) = 3$, $sup(t_8) = 3$, and
group C, $sup(t_2) = 6$, $sup(t_4) = 5$, $sup(t_9) = 4$

A		B		C	
IT1	t_1	IT2	t_5	IT1	t_2 t_4
IT2	t_1 t_3 t_6	IT3	t_8	IT3	t_2 t_4 t_9
IT3	t_6	IT4	t_5 t_7	IT5	t_2 t_4 t_9
IT4	t_3	IT5	t_8	IT6	t_2 t_4
IT6	t_1 t_3	IT7	t_5 t_7 t_8	IT7	t_2 t_9
IT8	t_3 t_6	IT8	t_7	IT8	t_2
				IT9	t_4 t_9

Table 16. Transaction containing items of A, B, C group

A		**B**		**C**	
Confidence	**Value**	**Confidence**	**Value**	**Confidence**	**Value**
$t_1 \to t_3$	0.66	$t_5 \to t_7$	0.66	$t_2 \to t_4$	0.66
$t_1 \to t_6$	0.66	$t_5 \to t_8$	0.33	$t_2 \to t_9$	0.5
$t_1 \to t_3 \cup t_6$	0.33	$t_5 \to t_7 \cup t_8$	0.33	$t_2 \to t_4 \cup t_9$	0.33
$t_3 \to t_1$	0.5	$t_7 \to t_5$	0.66	$t_4 \to t_2$	0.8
$t_3 \to t_6$	0.5	$t_7 \to t_8$	0.33	$t_4 \to t_9$	0.6
$t_3 \to t_1 \cup t_6$	0.25	$t_7 \to t_5 \cup t_8$	0.33	$t_4 \to t_2 \cup t_9$	0.4
$t_6 \to t_1$	0.33	$t_8 \to t_5$	0.33	$t_9 \to t_2$	0.75
$t_6 \to t_3$	0.66	$t_8 \to t_7$	0.33	$t_9 \to t_4$	0.75
$t_6 \to t_1 \cup t_3$	0.25	$t_8 \to t_5 \cup t_7$	0.33	$t_9 \to t_2 \cup t_4$	0.5

Table 17. Confidence of items group A, B, C

We used these confidence values and, calculated the **o**pportunity cost for items of each category are. Then index value are calculated in order to modify ordering policy for items by substituting values of opportunity cost in Eq. (5) and are given in Table 18.

Category	**Items**	**Opportunity cost**	**Index**
A	t_1	20.05	1.059053979
	t_3	18.75	1.039230485
	t_6	18.26	1.046822763
B	t_5	24.53	1.054565135
	t_7	24.49	1.035112428
	t_8	21.88	1.055584955
C	t_2	33.2	1.054965311
	t_4	38	1.032370803
	t_9	41.25	1.029857301

Table 18. Index values for groups of ABC classification

The optimal value of Z_{max}, $ETPU(Z)$, and EOQ of items of each of three classification group calculated as given by Table 19. A comparison between the proposed method and the state-of-art methods has been given in Table 20 and graphically shown in Fig.3.

Items	Z_{max}	$ETPU(Z)$	EOQ
t_1	765.933	65262.11714	811.1643912
t_3	1165.54	35621.61795	1211.264699
t_6	1102.31	125874.6077	1153.9232
t_5	521.191	46120.60722	549.6298573
t_7	770.318	37288.99159	797.3657353
t_8	501.421	27532.98501	529.2924637
t_2	292.728	20491.50061	308.8178856
t_4	296.26	18956.97161	305.8501741
t_9	292.897	13913.16909	301.6421139

Table 19. Modified ordering policy

Items	Traditional EOQ	EOQ (Mittal et al.)	EOQ (Present work)
t_1	748	808	811.1643912
t_5	505	543	549.6298573
t_2	283	304	308.8178856

Table 20. Comparison between the proposed model (with Classification) and existing models

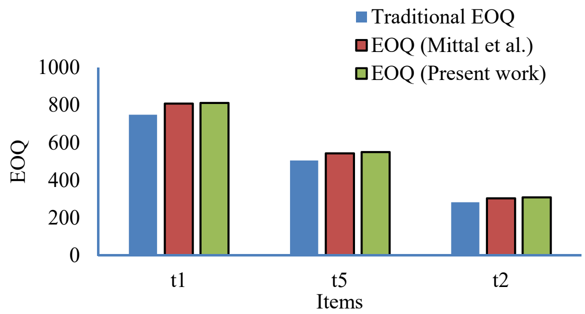

Fig. 3 Comparison of EOQ values obtained in proposed method (ABC classification) and existing methods.

6. Conclusion

In the present model, an investigation on the EOQ of inventory for defectives has been made after getting frequent item-set with the help of the EOQ formulae. It also introduces the allowable proportionate discounts according to the amount of imperfect products contained in received lot. In order to separate good and imperfect items from the lot a 100% screening is carried out. Later imperfect items are sold with a proportionate rate of discount. In previous section of this paper we have given numerical analysis. Three different examples were considered to illustrate the model with three different cases as apriori, clustering, and ABC classification respectively. Moreover, the idea is to find how cross-selling effects modify lot size, EOQ, and ETPU (total profit per unit) for imperfect items when merged proportionate discount and data mining techniques. Three dataset containing inventory transactions of different items are taken. In order to analyze the work done some features like various cost, selling price, annual demand, etc. are taken. However more number of association rules generated and chosen when clustering with Apriori algorithm considered for inventory transaction database that increases the total profit. In the case of ABC classification, we have taken every item from each category for analysis and compared ordering policies obtained. In each case of numerical example apriori algorithm is used to evaluate support and confidence. These values further used to find opportunity cost and modify EOQ.

Conflicts of interest: *The authors have no conflicts of interest to declare.*

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Notations

The following notations and assumptions have been used in this proposed work for the sake of convenience (Patro et al. (2017)) in model formulation:

Sl. No.	Abbreviation	Description
1	z	Ordering quantity size
2	D	demand year wise
3	C_p	cost unit wise
4	C_k	cost of ordering
5	C_h	carrying cost
6	OC_k	opportunity cost
7	I_{ndk}	Index
8	p	% of defectives in Z
9	$f(p)$	Probability Density Function of p
10	S_g	Unit selling price of a perfect item
11	S_{imp}	Price of selling for imperfect item
12	w	Rate of screening
13	C_s	Cost of screening of an item
14	T	Length of a cycle
15	TR	Total revenue of a cycle
16	TP	Total profit per cycle
17	TPU	Total profit in a unit time
18	$E[p]$	Expected % of defectives in z
19	$ETPU(Z)$	Expected total profit per unit time in z