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ENCRYPTING A WORD USING SUPER-EDGE ANTIMAGIC AND SUPER-EDGE MAGIC TOTAL LABELING OF EXTENDED DUPLICATE GRAPHS

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Abstract

Transmitting a secret word over web so that nobody interfere and interpret it is a challenging task and encrypting a word is a solution. In this paper, two encryption techniques that combine cryptographic algorithms like Affine-cipher and RSA algorithms with graph labeling algorithms are presented and illustrated. Super-edge antimagic total labeling for ladder graph's extended duplicate graph along with super-edge magic total labeling for comb graph's extended duplicate graph are being used in this article as graph labeling algorithms. A graph consisting μ nodes(vertices) with ϵ lines(edges) are labeled with first $\mu + \epsilon$ positive integers. Induced edge sum of edges are computed as addition of edge label with label of terminal vertices of that edge. If these induced edge sums are distinct for all edges, the labeling is termed as edge antimagic total labeling. Further to this, first nodes and then lines are labeled with consecutive positive integers, then it is termed as super-edge antimagic total labeling, whereas, in case all lines have same induced edge sum is a super-edge magic total labeling.

Keywords: **Affine-cipher algorithm, RSA algorithm, Ladder graph, Comb graph , Extended duplicate graph,Super-edge antimagic total labeling, Super-edge magic total labeling**

AMS classification: 05C78

1. Introduction

In-order to remove unwanted duplications , Sampathkumar [13] investigates on duplication of certain graphs on points and obtain a characterization on these graphs. R.Jegan et.al. [8,9] proved Super-edge magic and antimagic total labeling is being present in few extended duplicate graphs. William Stallings [17] gave his insights regarding the principles of Cryptography and Network security through his publishing , "Cryptography and Network security Principles and Practices". Manisha Kumari and Krishnanad [10] proposed a modified cipher algorithm that has been used to encrypt data that would provide better security while storing the data . I W. Sudarsana et.al [14] discussed on super mean and magic graphs labeling and their application for enhancing the shield levels of Affine Cipher for encoding words . Baizhu Ni et.al[3] proposed a few novel encryption algorithms applying corona graphs and bipartite graphs. Amudha.P [1] discussed a new algorithmic technique that uses a complete graph to encode and decode a message using graph labeling where anti-magic labeling is used.

2. Preliminaries

Definition 2.1: Definition of Ladder and Comb graphs

Consider a path graph P_μ with μ nodes. The Cartesian product of P_μ with P_2 is termed as Ladder graph L_μ. A Ladder graph L_μ consists of 2μ nodes and $3\mu - 2$ lines.
The graph resulted from P_μ by adjoining a unique node (different from already existing nodes) with each node in the path is a comb graph C_μ. A comb graph consists of 2μ nodes and $2\mu - 1$ lines. .

Definition 2.2: Definition of Duplicate graph

Consider a graph G consisting ν nodes with ϵ lines. Its duplicate graph DG is derived using the following steps:
(i) a new node set V' disjoint from V having same number of nodes V is considered and each node α in V is associated with a node (vertex) α' in V'
(ii) Duplicate graph has node set $V \cup V'$
(iii) Each line $\alpha\beta$ in the graph G generates two lines $\alpha\beta'$ and $\alpha'\beta$ in DG.

Note: Duplicate graph of Ladder graph consists of 4μ nodes with $6\mu - 4$ lines. Lines of Ladder graph's duplicate graph shall be taken as $\alpha_{2\,\kappa-1}\alpha'_{2\,\kappa}, \alpha'_{2\,\kappa-1}\alpha_{2\,\kappa}\ (for\ \ \kappa = 1\ to\ \mu)$
$\alpha_{2\,\kappa-1}\alpha'_{2\,\kappa+1}, \alpha'_{2\,\kappa-1}\alpha_{2\,\kappa+1}, \alpha_{2\,\kappa}\alpha'_{2\,\kappa+2}, \alpha'_{2\,\kappa}\alpha_{2\,\kappa+2}\big(\,(for\ \kappa = 1\ to\ \mu - 1)\big)$.
Comb graph's duplicate graph consists of 4μ nodes and $4\mu - 2$ lines. Lines of Comb graph's duplicate graph of shall be taken as $\alpha_{2\,\kappa-1}\alpha'_{2\,\kappa}, \alpha'_{2\,\kappa-1}\alpha_{2\,\kappa}\ (for\ \ \kappa = 1\ to\ \mu)$
$\alpha_{2\,\kappa-1}\alpha'_{2\,\kappa+1}, \alpha'_{2\,\kappa-1}\alpha_{2\,\kappa+1}, (for\ \kappa = 1\ to\ \mu - 1)$

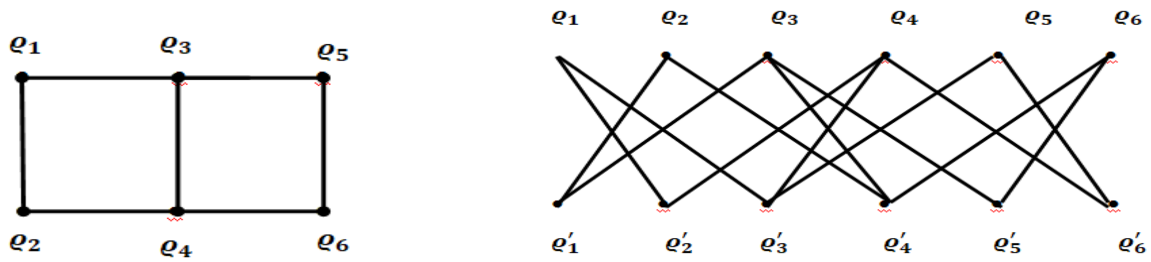

Fig 1: Ladder graph L_3 and its duplicate graph $DG(L_3)$

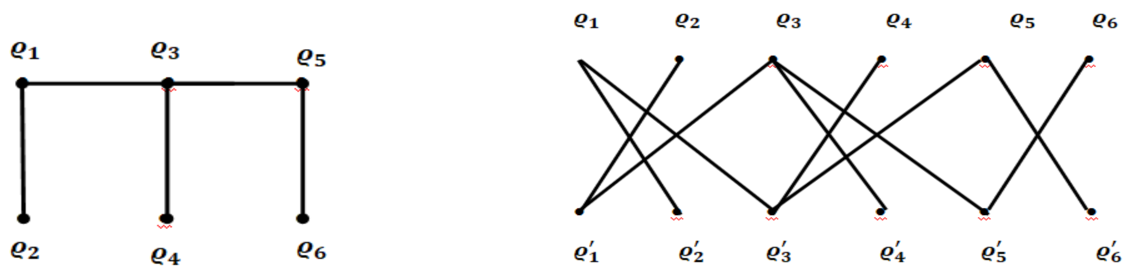

Fig 2: Comb graph CB_3 and its duplicate graph $DG(CB_3)$

In case, the duplicate graph is disconnected then two arbitrary nodes $\alpha_\kappa, \alpha'_\kappa$ preferably other than first and last nodes are connected by a line to make the duplicate graph connected and the graph thus obtained is termed as extended duplicate graph EDG.

Definition 2.3: Definition of Super-edge antimagic total labeling and Super-edge magic total labeling

Consider G consisting ν nodes with ϵ lines. A super-edge antimagic total labeling is a function $\varphi: G = (V, E) \rightarrow \{1,2,3, \dots, \nu + \epsilon\}$ in which nodes are labeled with integers 1,2,3,. ν and lines are labeled with integers $\nu + 1, \nu + 2, \dots, \nu + \epsilon$ satisfying the condition that induced edge sum φ^* of a line is defined as $\varphi^*(\alpha\beta) = \varphi(\alpha) + \varphi(\beta) + \varphi(\alpha\beta)$ takes different values on different edges. On the other hand, if these induced edge sums are all same for all the lines, then it is termed as super-edge magic total labeling.

Theorem 2.4: $EDG(L_{13})$ **acknowledges super-edge antimagic total labeling.**

Proof:

nodes	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5	ϱ_6	ϱ_7	ϱ_8	ϱ_9	ϱ_{10}	ϱ_{11}	ϱ_{12}	ϱ_{13}
label	26	39	24	11	28	41	22	9	43	20	7	32	45
nodes	ϱ_{14}	ϱ_{15}	ϱ_{16}	ϱ_{17}	ϱ_{18}	ϱ_{19}	ϱ_{20}	ϱ_{21}	ϱ_{22}	ϱ_{23}	ϱ_{24}	ϱ_{25}	ϱ_{26}
label	45	18	5	34	47	16	3	36	49	14	1	38	51
nodes	ϱ'_1	ϱ'_2	ϱ'_3	ϱ'_4	ϱ'_5	ϱ'_6	ϱ'_7	ϱ'_8	ϱ'_9	ϱ'_{10}	ϱ'_{11}	ϱ'_{12}	ϱ'_{13}
label	25	40	23	12	27	42	21	10	29	44	19	8	31
nodes	ϱ'_{14}	ϱ'_{15}	ϱ'_{16}	ϱ'_{17}	ϱ'_{18}	ϱ'_{19}	ϱ'_{20}	ϱ'_{21}	ϱ'_{22}	ϱ'_{23}	ϱ'_{24}	ϱ'_{25}	ϱ'_{26}
label	46	17	6	33	48	15	4	35	50	13	2	37	52

Table 1: Node-labeling for super-edge antimagic total labeling of $EDG(L_{13})$

lines	$\varrho_1\varrho'_2$	$\varrho_1\varrho'_3$	$\varrho_2\varrho'_4$	$\varrho_3\varrho'_4$	$\varrho_3\varrho'_5$	$\varrho_4\varrho'_6$	$\varrho_5\varrho'_6$	$\varrho_5\varrho'_7$	$\varrho_6\varrho'_8$	$\varrho_7\varrho'_8$
label	11	65	78	64	89	101	116	67	80	62
lines	$\varrho_7\varrho'_9$	$\varrho_8\varrho'_{10}$	$\varrho_9\varrho'_{10}$	$\varrho_9\varrho'_{11}$	$\varrho_{10}\varrho'_{12}$	$\varrho_{11}\varrho'_{12}$	$\varrho_{11}\varrho'_{13}$	$\varrho_{12}\varrho'_{14}$	$\varrho_{13}\varrho'_{14}$	
label	91	103	118	69	82	60	93	105	120	
lines	$\varrho'_1\varrho_2$	$\varrho'_1\varrho_3$	$\varrho'_2\varrho_4$	$\varrho'_3\varrho_4$	$\varrho'_3\varrho_5$	$\varrho'_4\varrho_6$	$\varrho'_5\varrho_6$	$\varrho'_5\varrho_7$	$\varrho'_6\varrho_8$	$\varrho'_7\varrho_8$
label	113	66	77	63	90	102	115	68	79	61
edges	$\varrho'_7\varrho_9$	$\varrho'_8\varrho_{10}$	$\varrho'_9\varrho_{10}$	$\varrho'_9\varrho_{11}$	$\varrho'_{10}\varrho_{12}$	$\varrho'_{11}\varrho_{12}$	$\varrho'_{11}\varrho_{13}$	$\varrho'_{12}\varrho_{14}$	$\varrho'_{13}\varrho_{14}$	
label	92	104	117	70	81	59	94	106	119	
edges	$\varrho_{13}\varrho'_{15}$	$\varrho_{14}\varrho'_{16}$	$\varrho_{15}\varrho'_{16}$	$\varrho_{15}\varrho'_{17}$	$\varrho_{16}\varrho'_{18}$	$\varrho_{17}\varrho'_{18}$	$\varrho_{17}\varrho'_{19}$	$\varrho_{18}\varrho'_{20}$	$\varrho_{19}\varrho'_{20}$	
label	71	84	58	95	107	122	73	86	56	
edges	$\varrho'_{13}\varrho_{14}$	$\varrho'_{14}\varrho_{16}$	$\varrho'_{15}\varrho_{16}$	$\varrho'_{15}\varrho_{17}$	$\varrho'_{16}\varrho_{18}$	$\varrho'_{17}\varrho_{18}$	$\varrho'_{17}\varrho_{19}$	$\varrho'_{18}\varrho_{20}$	$\varrho'_{19}\varrho_{20}$	
label	119	83	57	96	108	121	74	85	55	
edges	$\varrho_{19}\varrho'_{21}$	$\varrho_{20}\varrho'_{22}$	$\varrho_{21}\varrho'_{22}$	$\varrho_{21}\varrho'_{23}$	$\varrho_{22}\varrho'_{24}$	$\varrho_{23}\varrho'_{24}$	$\varrho_{23}\varrho'_{25}$	$\varrho_{24}\varrho'_{26}$	$\varrho_{25}\varrho'_{26}$	
label	97	109	124	75	88	57	99	111	126	
edges	$\varrho'_{19}\varrho_{21}$	$\varrho'_{20}\varrho_{22}$	$\varrho'_{21}\varrho_{22}$	$\varrho'_{21}\varrho_{23}$	$\varrho'_{22}\varrho_{24}$	$\varrho'_{23}\varrho_{24}$	$\varrho'_{23}\varrho_{25}$	$\varrho'_{24}\varrho_{26}$	$\varrho'_{25}\varrho_{26}$	$\varrho_{25}\varrho'_{25}$
label	55	110	123	76	87	53	100	112	125	127

Table 2: Line-labeling of super-edge antimagic total labeling of $EDG(L_{13})$

Verifying that this labeling satisfies the criteria in definition 2.3 and hence $EDG(L_{13})$ acknowledges super-edge antimagic total labeling is a trivial one.

Theorem 2.5: $EDG(CB_{14})$ **permits super-edge magic total labeling.**

Proof:

nodes	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5	ϱ_6	ϱ_7	ϱ_8	ϱ_9	ϱ_{10}	ϱ_{11}	ϱ_{12}	ϱ_{13}	ϱ_{14}
label	28	1	2	27	26	3	4	25	24	5	6	23	22	7
nodes	ϱ_{15}	ϱ_{16}	ϱ_{17}	ϱ_{18}	ϱ_{19}	ϱ_{20}	ϱ_{21}	ϱ_{22}	ϱ_{23}	ϱ_{24}	ϱ_{25}	ϱ_{26}	ϱ_{27}	ϱ_{28}
label	8	21	20	9	10	19	18	11	12	17	16	13	14	15
nodes	ϱ'_1	ϱ'_2	ϱ'_3	ϱ'_4	ϱ'_5	ϱ'_6	ϱ'_7	ϱ'_8	ϱ'_9	ϱ'_{10}	ϱ'_{11}	ϱ'_{12}	ϱ'_{13}	ϱ'_{14}
label	29	56	55	30	31	54	53	32	33	52	51	34	35	50
nodes	ϱ'_{15}	ϱ'_{16}	ϱ'_{17}	ϱ'_{18}	ϱ'_{19}	ϱ'_{20}	ϱ'_{21}	ϱ'_{22}	ϱ'_{23}	ϱ'_{24}	ϱ'_{25}	ϱ'_{26}	ϱ'_{27}	ϱ'_{28}
label	49	36	37	48	47	38	39	46	45	40	41	44	43	42

Table 3: Node-labeling for super-edge magic total labeling of $EDG(CB_{14})$

lines	$\varrho_1\varrho'_2$	$\varrho_1\varrho'_3$	$\varrho_3\varrho'_4$	$\varrho_3\varrho'_5$	$\varrho_5\varrho'_6$	$\varrho_5\varrho'_7$	$\varrho_7\varrho'_8$	$\varrho_7\varrho'_9$	$\varrho_9\varrho'_{10}$	$\varrho_9\varrho'_{11}$
label	57	58	109	108	61	62	105	104	65	66
lines	$\varrho'_1\varrho_2$	$\varrho'_1\varrho_3$	$\varrho'_3\varrho_4$	$\varrho'_3\varrho_5$	$\varrho'_5\varrho_6$	$\varrho'_5\varrho_7$	$\varrho'_7\varrho_8$	$\varrho'_7\varrho_9$	$\varrho'_9\varrho_{10}$	$\varrho'_9\varrho_{11}$
label	111	110	59	60	107	106	63	64	103	102
lines	$\varrho_{11}\varrho'_{12}$	$\varrho_{11}\varrho'_{13}$	$\varrho_{13}\varrho'_{14}$	$\varrho_{13}\varrho'_{15}$	$\varrho_{15}\varrho'_{16}$	$\varrho_{15}\varrho'_{17}$	$\varrho_{17}\varrho'_{18}$	$\varrho_{17}\varrho'_{19}$	$\varrho_{19}\varrho'_{20}$	$\varrho_{19}\varrho'_{21}$
label	101	100	69	70	97	96	73	74	93	94
lines	$\varrho'_{11}\varrho_{12}$	$\varrho'_{11}\varrho_{13}$	$\varrho'_{13}\varrho_{14}$	$\varrho'_{13}\varrho_{15}$	$\varrho'_{15}\varrho_{16}$	$\varrho'_{15}\varrho_{17}$	$\varrho'_{17}\varrho_{18}$	$\varrho'_{17}\varrho_{19}$	$\varrho'_{19}\varrho_{20}$	$\varrho'_{19}\varrho_{21}$
label	67	68	99	98	71	72	95	94	75	76
lines	$\varrho_{21}\varrho'_{22}$	$\varrho_{21}\varrho'_{23}$	$\varrho_{23}\varrho'_{24}$	$\varrho_{23}\varrho'_{25}$	$\varrho_{25}\varrho'_{26}$	$\varrho_{25}\varrho'_{27}$	$\varrho_{27}\varrho'_{28}$			
label	77	78	89	88	81	82	85			
lines	$\varrho'_{21}\varrho_{22}$	$\varrho'_{21}\varrho_{23}$	$\varrho'_{23}\varrho_{24}$	$\varrho'_{23}\varrho_{25}$	$\varrho'_{25}\varrho_{26}$	$\varrho'_{25}\varrho_{27}$	$\varrho'_{27}\varrho_{28}$	$\varrho_{27}\varrho'_{27}$		
label	91	90	79	80	87	86	83	84		

Table 4: Line-labeling for super-edge magic total labeling of $EDG(CB_{14})$

Verifying the criteria of definition 2.3 are satisfied with magic total 141 and hence $EDG(CB_{14})$ permits super-edge magic total labeling is typical.

3. Main Results

Now let us see two algorithms to encrypt a word into sequence of numbers and then present algorithms to decrypt the sequence of numbers to original word.

Encryption algorithm 3.1

1. Associate each alphabet with a unique vertex of $EDG(L_{13})$ as in Table 5.

nodes	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5	ϱ_6	ϱ_7	ϱ_8	ϱ_9	ϱ_{10}	ϱ_{11}	ϱ_{12}	ϱ_{13}
alphabet	A	B	C	D	E	F	G	H	I	J	K	L	M
nodes	ϱ_{14}	ϱ_{15}	ϱ_{16}	ϱ_{17}	ϱ_{18}	ϱ_{19}	ϱ_{20}	ϱ_{21}	ϱ_{22}	ϱ_{23}	ϱ_{24}	ϱ_{25}	ϱ_{26}
alphabet	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
nodes	ϱ'_1	ϱ'_2	ϱ'_3	ϱ'_4	ϱ'_5	ϱ'_6	ϱ'_7	ϱ'_8	ϱ'_9	ϱ'_{10}	ϱ'_{11}	ϱ'_{12}	ϱ'_{13}
alphabet	a	b	c	d	e	f	g	h	i	j	k	l	m
nodes	ϱ'_{14}	ϱ'_{15}	ϱ'_{16}	ϱ'_{17}	ϱ'_{18}	ϱ'_{19}	ϱ'_{20}	ϱ'_{21}	ϱ'_{22}	ϱ'_{23}	ϱ'_{24}	ϱ'_{25}	ϱ'_{26}
alphabet	n	o	p	q	r	s	t	u	v	w	x	y	z

Table 5: Association between vertices of $EDG(L_{13})$ and alphabets.

2. Consider the secret word to be encrypted as $\omega = \lambda_1\lambda_2 \dots \lambda_n$ where λ_κ is the i^{th} character of the word. Define an one-one function ψ for each character in the secret word as

$$\psi(\lambda_\kappa) = \begin{cases} \varphi(\varrho_\iota\varrho'_{\iota+2}), & \lambda_\kappa \text{ is upper case alphabet} \\ \varphi(\varrho'_\iota\alpha\varrho_{\iota+1}), & \lambda_\kappa \text{ is lower case alphabet}, \iota \text{ is odd} \\ \varphi(\varrho'_\iota\varrho_{\iota-1}), & \lambda_\kappa \text{ is lower case alphabet}, \iota \text{ is even} \end{cases}$$

Where ϱ_ι or ϱ'_ι is the node associated with alphabet λ_κ in step 1 for alphabets other than Y *and* Z.

3. $\psi(Y) = \varphi(\varrho_{25}\varrho'_{25})$; $\psi(Z) = \varphi(\varrho_{26}\varrho'_{24})$
4. Select two integers a satisfying $1 < a < 104$ *and* $gcd(a, 104) = 1$ and b satisfying $0 \leq b < 104$
5. Take $\rho_\kappa = a\psi(\lambda_\kappa) + b$
6. Encrypted sequence of numbers is $\rho_1\rho_2 \dots \rho_n$

Illustration 3.2

Consider the secret word **DisCretE**

The vertices associated with the characters of this word are

$$\varrho_4 \quad \varrho'_9 \quad \varrho'_{19} \quad \varrho_3 \quad \varrho'_{18} \quad \varrho'_5 \quad \varrho'_{20} \quad \varrho_5$$

The associated edges are

$\varrho_4\varrho'_6$ $\varrho'_9\varrho_{10}$ $\varrho'_{19}\varrho_{20}$ $\varrho_3\varrho'_5$ $\varrho_{17}\varrho'_{18}$ $\varrho'_5\varrho_6$ $\varrho_{19}\varrho'_{20}$ $\varrho_5\varrho'_7$

The respective edge labels are

$\varphi(\varrho_4\,\varrho'_6\,) = 101$ $\varphi(\varrho'_9\,\varrho_{10}) = 117$ $\varphi(\varrho'_{19}\varrho_{20}) = 55$ $\varphi(\varrho_3\varrho'_5) = 89$
$\varphi(\varrho_{17}\varrho'_{18}) = 122$ $\varphi(\varrho'_5\varrho_6) = 115$ $\varphi(\varrho_{19}\varrho'_{20}) = 56$ $\varphi(\varrho_5\varrho'_7) = 67$
$\psi(\mathrm{D}) = 101$ $\psi(\mathrm{i}) = 117$ $\psi(\mathrm{s}) = 55$ $\psi(\mathrm{C}) = 89$ $\psi(\mathrm{r}) = 122$
$\psi(\mathrm{e}) = 115$ $\psi(\mathrm{t}) = 56$ $\psi(\mathrm{E}) = 67$
Applying affine-cipher with $a = 21$ $b = 2$ with respect to $mod\ 104$ we get
$\rho_1 = 43$ $\rho_2 = 67$ $\rho_3 = 13$ $\rho_4 = 103$ $\rho_5 = 68$
$\rho_6 = 25$ $\rho_7 = 34$ $\rho_8 = 57$

Decryption algorithm 3.3

Consider an encrypted sequence $\rho_1\rho_2 \ldots \rho_n$

1. Find $a^{-1}(mod\ 104)$.
2. Find $\psi(\lambda_\kappa) = a^{-1}\rho_\kappa - b$
3. Only if $\psi(\lambda_\kappa) < 52$, then take $\psi(\lambda_\kappa) = \psi(\lambda_\kappa) + 104$
4. Find the edge $\alpha_\iota\alpha'_\tau$with $\psi(\lambda_\kappa) = \varphi(\varrho_\iota\varrho'_\tau)$
5. If $|\tau - \iota|$=1, then take λ_κ = vertex corresponding to ϱ'_τ in lower case
6. If $|\tau - \iota|$=2, then take λ_κ = vertex corresponding to ϱ_ι in upper case
7. Decrypted word is $\omega = \lambda_1\lambda_2 \ldots \lambda_n$.

Illustration 3.4

$\rho_1 = 43$ $\rho_2 = 67$ $\rho_3 = 13$ $\rho_4 = 103$ $\rho_5 = 25$ $\rho_6 = 34$ $\rho_7 = 57$
Applying affine-cipher with $a = 21$ $a^{-1} = 5$ $b = 2$ $with\ mod\ 104$
$\psi(\lambda_1) = 101$ $\psi(\lambda_2) = 13$ $\psi(\lambda_3) = 55$ $\psi(\lambda_4) = 89$ $\psi(\lambda_5) = 18$
$\psi(\lambda_6) = 11$ $\psi(\lambda_7) = 56$ $\psi(\lambda_8) = 67$
Modify above numbers if less than 52
$\psi(\lambda_1) = 101$ $\psi(\lambda_2) = 117$ $\psi(\lambda_3) = 55$ $\psi(\lambda_4) = 89$ $\psi(\lambda_5) = 122$
$\psi(\lambda_6) = 115$ $\psi(\lambda_7) = 56$ $\psi(\lambda_8) = 67$
The edges corresponding to these labels are
$\varphi(\varrho_4\,\varrho'_6\,) = 101$ $\varphi(\varrho'_9\,\varrho_{10}) = 117$ $\varphi(\varrho'_{19}\varrho_{20}) = 55$ $\varphi(\varrho_3\varrho'_5) = 89$
$\varphi(\varrho_{17}\varrho'_{18}) = 122$ $\varphi(\varrho'_5\varrho_6) = 115$ $\varphi(\varrho_{19}\varrho'_{20}) = 56$ $\varphi(\varrho_5\varrho'_7) = 67$
Considering the difference of suffices, the corresponding vertices are
ϱ_4 ϱ'_9 ϱ'_{19} ϱ_3 ϱ'_{18} ϱ'_5 ϱ'_{20} ϱ_5
The associated alphabets are DisCretE which is the secret word.

Encryption algorithm 3.5

1. Associate each alphabet with a unique vertex in $EDG(CB_{14})$ as in Table 5. Note that the vertices $\varrho_{27}, \varrho_{28}, \varrho'_{27}, \varrho'_{28}$ are dummy vertices associated with no alphabets.
2. Consider the secret word to be encrypted as $\omega = \lambda_1\lambda_2 \ldots \lambda_n$ where λ_κ is the κ^{th} character of the word.
3. Apply super-edge magic total labeling ϕ for $EDG(CB_{14})$
4. Define an one-one function ψ for each character in the secret word as

$$\psi(\lambda_\kappa) = \begin{cases} \phi(\varrho_\iota\varrho'_{\iota+2}), & \lambda_\kappa\ is\ upper\ case\ alphabet, \iota\ is\ odd \\ \phi(\varrho_\iota\varrho'_{\iota-1}), & \lambda_\kappa\ is\ upper\ case\ alphabet, \iota\ is\ even \\ \phi(\varrho'_\iota\varrho_{\iota+2}), & \lambda_\kappa\ is\ lower\ case\ alphabet, \iota\ is\ odd \\ \phi(\varrho'_\iota\varrho_{\iota-1}), & \lambda_\kappa\ is\ lower\ case\ alphabet, \iota\ is\ even \end{cases},$$

Where ϱ_ι or ϱ'_ι is the vertex associated with alphabet λ_κ.
5. Encrypt $\psi(\lambda_\kappa)$ with public key e of RSA algorithm $\rho_\kappa = \psi(\lambda_\kappa)^e\ \ mod\ \ n$
where $n > 111$ and the encrypted sequence of numbers is $\rho_1\rho_2 \ldots \rho_n$

Illustration 3.6

Consider the secret word$\omega = ComPuTEr$.
Assign super-edge magic total labeling ϕ for $EDG(CB_{14})$
Alphabet-vertices association is given below:
C:ϱ_3 o:ϱ'_{15} m: ϱ'_{13} P: ϱ_{16} u: ϱ'_{21} T: ϱ_{20} E: ϱ_5 r: ϱ'_{18}
Then

$\psi(\lambda_1) = \phi(\varrho_3\varrho_5') = 108$ $\psi(\lambda_2) = \phi(\varrho_{15}'\varrho_{17}) = 72$ $\psi(\lambda_3) = \phi(\varrho_{13}'\varrho_{15}) = 98$
$\psi(\lambda_4) = \phi(\varrho_{16}\varrho_{15}') = 71$
$\psi(\lambda_5) = \phi(\varrho_{21}'\varrho_{23}) = 90$ $\psi(\lambda_6) = \phi(\varrho_{20}\varrho_{19}') = 75$ $\psi(\lambda_7) = \phi(\varrho_5\varrho_7') = 62$
$\psi(\lambda_8) = \phi(\varrho_{18}'\varrho_{17}) = 73$
Choose prime numbers 7,17 $n = 119$ $\varphi(119) = 96$ public key $e = 11$ for applying RSA algorithm
$\rho_1 = 108^{11} mod 119 = 5$ $\rho_2 = 72^{11} mod\ 119 = 81$
$\rho_3 = 98^{11} mod 119 = 21$ $\rho_4 = 71^{11} mod\ 119 = 92$
$\rho_5 = 90^{11} mod 119 = 62$ $\rho_6 = 75^{11} mod\ 119 = 31$
$\rho_7 = 62^{11} mod 119 = 97$ $\rho_8 = 73^{11} mod\ 119 = 96$
The encrypted sequence is **5 81 21 92 62 31 97 96**

Decryption algorithm 3.7

1. Consider the encrypted sequence $\rho_1\rho_2 \dots \rho_n$
2. Decrypt the sequence with private key d corresponding to the public key e of the RSA algorithm $\psi(\lambda_i) = \rho_i{}^d \ mod\ n$
3. Find the corresponding edge $\alpha_\iota\alpha_k'$ with label $\psi(\lambda_i)$
4. (i) If $|\iota - \tau| = 2$,$\iota > \tau$ then λ_κ is the alphabet associated with node ϱ_τ'

(ii) If $|\iota - \tau| = 2$, $\iota < \tau$ then λ_κ is the alphabet associated with node ϱ_ι
(iii)If $|\iota - \tau| = 1$, $\iota > \tau$ then λ_κ is the alphabet associated with node ϱ_ι
(iv)If $|\iota - \tau| = 1$, $\iota < \tau$ then λ_κ is the alphabet associated with node ϱ_τ'

5. Take the alphabets associated with these nodes from Table 5.
6. The decrypted word will be obtained.

Illustration 3.8

Consider the sequence of numbers **5 81 21 92 62 31 97 96**

Decrypt the sequence with private key =35 for the public key = 11 with prime numbers 7,17 for applying RSA algorithm
The decrypted sequence is 108 72 98 71 90 75 62 73
Corresponding edges in $EDG(CB_{14})$ with these labels are
$\varrho_3\varrho_5'$ $\varrho_{15}'\varrho_{17}$ $\varrho_{13}'\varrho_{15}$ $\varrho_{15}'\varrho_{16}$ $\varrho_{21}'\varrho_{23}$ $\varrho_{19}'\varrho_{20}$ $\varrho_5 v_7'$ $\varrho_{17}\varrho_{18}'$
Associated vertices are ϱ_3 ϱ_{15}' ϱ_{13}' ϱ_{16} ϱ_{21}' ϱ_{20} ϱ_5 ϱ_{18}'
From Table 5 the alphabets associated with these vertices are C o m P u T E r

Conclusion and future work

Encryption algorithms involving Affine-cipher and RSA algorithms combining some EDG's Super-edge magic and antimagic total labeling are presented and verified with examples. JAVA coding for the same algorithms may be written and their performance and complexity may be studied in future to enhance these algorithms.

Conflicts of Interest: Authors have no conflicts of interest to declare.

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