$\varphi(\varrho_4\,\varrho_6') = 101$ $\varphi(\varrho_9'\,\varrho_{10}) = 117$ $\varphi(\varrho_{19}'\varrho_{20}) = 55$ $\varphi(\varrho_3\varrho_5') = 89$
$\varphi(\varrho_{17}\varrho_{18}') = 122$ $\varphi(\varrho_5'\varrho_6) = 115$ $\varphi(\varrho_{19}\varrho_{20}') = 56$ $\varphi(\varrho_5\varrho_7') = 67$
$\psi(\mathrm{D}) = 101$ $\psi(\mathrm{i}) = 117$ $\psi(\mathrm{s}) = 55$ $\psi(\mathrm{C}) = 89$ $\psi(\mathrm{r}) = 122$
$\psi(\mathrm{e}) = 115$ $\psi(\mathrm{t}) = 56$ $\psi(\mathrm{E}) = 67$
Applying affine-cipher with $a = 21$ $b = 2$ with respect to mod 104 we get
$\rho_1 = 43$ $\rho_2 = 67$ $\rho_3 = 13$ $\rho_4 = 103$ $\rho_5 = 68$
$\rho_6 = 25$ $\rho_7 = 34$ $\rho_8 = 57$

Decryption algorithm 3.3

Consider an encrypted sequence $\rho_1\rho_2 \dots \rho_n$

1. Find $a^{-1}(mod\ 104)$.
2. Find $\psi(\lambda_\kappa) = a^{-1}\rho_\kappa - b$
3. Only if $\psi(\lambda_\kappa) < 52$, then take $\psi(\lambda_\kappa) = \psi(\lambda_\kappa) + 104$
4. Find the edge $\alpha_\iota\alpha_\tau'$with $\psi(\lambda_\kappa) = \varphi(\varrho_\iota\varrho_\tau')$
5. If $|\tau - \iota|$=1, then take λ_κ = vertex corresponding to ϱ_τ' in lower case
6. If $|\tau - \iota|$=2, then take λ_κ = vertex corresponding to ϱ_ι in upper case
7. Decrypted word is $\omega = \lambda_1\lambda_2 \dots \lambda_n$.

Illustration 3.4

$\rho_1 = 43$ $\rho_2 = 67$ $\rho_3 = 13$ $\rho_4 = 103$ $\rho_5 = 25$ $\rho_6 = 34$ $\rho_7 = 57$
Applying affine-cipher with $a = 21$ $a^{-1} = 5$ $b = 2$ $with\ mod$ 104
$\psi(\lambda_1) = 101$ $\psi(\lambda_2) = 13$ $\psi(\lambda_3) = 55$ $\psi(\lambda_4) = 89$ $\psi(\lambda_5) = 18$
$\psi(\lambda_6) = 11$ $\psi(\lambda_7) = 56$ $\psi(\lambda_8) = 67$
Modify above numbers if less than 52
$\psi(\lambda_1) = 101$ $\psi(\lambda_2) = 117$ $\psi(\lambda_3) = 55$ $\psi(\lambda_4) = 89$ $\psi(\lambda_5) = 122$
$\psi(\lambda_6) = 115$ $\psi(\lambda_7) = 56$ $\psi(\lambda_8) = 67$
The edges corresponding to these labels are
$\varphi(\varrho_4\,\varrho_6') = 101$ $\varphi(\varrho_9'\,\varrho_{10}) = 117$ $\varphi(\varrho_{19}'\varrho_{20}) = 55$ $\varphi(\varrho_3\varrho_5') = 89$
$\varphi(\varrho_{17}\varrho_{18}') = 122$ $\varphi(\varrho_5'\varrho_6) = 115$ $\varphi(\varrho_{19}\varrho_{20}') = 56$ $\varphi(\varrho_5\varrho_7') = 67$
Considering the difference of suffices, the corresponding vertices are
ϱ_4 ϱ_9' ϱ_{19}' ϱ_3 ϱ_{18}' ϱ_5' ϱ_{20}' ϱ_5
The associated alphabets are DisCretE which is the secret word.

Encryption algorithm 3.5

1. Associate each alphabet with a unique vertex in $EDG(CB_{14})$ as in Table 5. Note that the vertices $\varrho_{27}, \varrho_{28}, \varrho_{27}', \varrho_{28}'$ are dummy vertices associated with no alphabets.
2. Consider the secret word to be encrypted as $\omega = \lambda_1\lambda_2 \dots \lambda_n$ where λ_κ is the κ^{th} character of the word.
3. Apply super-edge magic total labeling ϕ for $EDG(CB_{14})$
4. Define an one-one function ψ for each character in the secret word as

$$\psi(\lambda_\kappa) = \begin{cases} \phi(\varrho_\iota\varrho_{\iota+2}'), & \lambda_\kappa\ is\ upper\ case\ alphabet, \iota\ is\ odd \\ \phi(\varrho_\iota\varrho_{\iota-1}'), & \lambda_\kappa\ is\ upper\ case\ alphabet, \iota\ is\ even \\ \phi(\varrho_\iota'\varrho_{\iota+2}), & \lambda_\kappa\ is\ lower\ case\ alphabet, \iota\ is\ odd \\ \phi(\varrho_\iota'\varrho_{\iota-1}), & \lambda_\kappa\ is\ lower\ case\ alphabet, \iota\ is\ even \end{cases},$$

Where ϱ_ι or ϱ_ι' is the vertex associated with alphabet λ_κ.
5. Encrypt $\psi(\lambda_\kappa)$ with public key e of RSA algorithm $\rho_\kappa = \psi(\lambda_\kappa)^e \ mod\ n$
where $n > 111$ and the encrypted sequence of numbers is $\rho_1\rho_2 \dots \rho_n$

Illustration 3.6

Consider the secret word$\omega = ComPuTEr$.
Assign super-edge magic total labeling ϕ for $EDG(CB_{14})$
Alphabet-vertices association is given below:
C:ϱ_3 o:ϱ_{15}' m: ϱ_{13}' P: ϱ_{16} u: ϱ_{21}' T: ϱ_{20} E: ϱ_5 r: ϱ_{18}'
Then

$\psi(\lambda_1) = \phi(\varrho_3\varrho_5') = 108$ $\psi(\lambda_2) = \phi(\varrho_{15}'\varrho_{17}) = 72$ $\psi(\lambda_3) = \phi(\varrho_{13}'\varrho_{15}) = 98$
$\psi(\lambda_4) = \phi(\varrho_{16}\varrho_{15}') = 71$
$\psi(\lambda_5) = \phi(\varrho_{21}'\varrho_{23}) = 90$ $\psi(\lambda_6) = \phi(\varrho_{20}\varrho_{19}') = 75$ $\psi(\lambda_7) = \phi(\varrho_5\varrho_7') = 62$
$\psi(\lambda_8) = \phi(\varrho_{18}'\varrho_{17}) = 73$
Choose prime numbers 7,17 $n = 119$ $\varphi(119) = 96$ public key $e = 11$ for applying RSA algorithm
$\rho_1 = 108^{11} mod 119 = 5$ $\rho_2 = 72^{11} mod\ 119 = 81$
$\rho_3 = 98^{11} mod 119 = 21$ $\rho_4 = 71^{11} mod\ 119 = 92$
$\rho_5 = 90^{11} mod 119 = 62$ $\rho_6 = 75^{11} mod\ 119 = 31$
$\rho_7 = 62^{11} mod 119 = 97$ $\rho_8 = 73^{11} mod\ 119 = 96$
The encrypted sequence is **5 81 21 92 62 31 97 96**

Decryption algorithm 3.7

1. Consider the encrypted sequence $\rho_1\rho_2 \ldots \rho_n$
2. Decrypt the sequence with private key d corresponding to the public key e of the RSA algorithm $\psi(\lambda_i) = \rho_i{}^d \ mod\ n$
3. Find the corresponding edge $\alpha_\iota\alpha_k'$ with label $\psi(\lambda_i)$
4. (i) If $|\iota - \tau| = 2$,$\iota > \tau$ then λ_κ is the alphabet associated with node ϱ_τ'

(ii) If $|\iota - \tau| = 2$, $\iota < \tau$ then λ_κ is the alphabet associated with node ϱ_ι
(iii)If $|\iota - \tau| = 1$, $\iota > \tau$ then λ_κ is the alphabet associated with node ϱ_ι
(iv)If $|\iota - \tau| = 1$, $\iota < \tau$ then λ_κ is the alphabet associated with node ϱ_τ'

5. Take the alphabets associated with these nodes from Table 5.
6. The decrypted word will be obtained.

Illustration 3.8

Consider the sequence of numbers **5 81 21 92 62 31 97 96**

Decrypt the sequence with private key =35 for the public key = 11 with prime numbers 7,17 for applying RSA algorithm
The decrypted sequence is 108 72 98 71 90 75 62 73
Corresponding edges in $EDG(CB_{14})$ with these labels are
$\varrho_3\varrho_5'$ $\varrho_{15}'\varrho_{17}$ $\varrho_{13}'\varrho_{15}$ $\varrho_{15}'\varrho_{16}$ $\varrho_{21}'\varrho_{23}$ $\varrho_{19}'\varrho_{20}$ $\varrho_5 v_7'$ $\varrho_{17}\varrho_{18}'$
Associated vertices are ϱ_3 ϱ_{15}' ϱ_{13}' ϱ_{16} ϱ_{21}' ϱ_{20} ϱ_5 ϱ_{18}'
From Table 5 the alphabets associated with these vertices are C o m P u T E r

Conclusion and future work

Encryption algorithms involving Affine-cipher and RSA algorithms combining some EDG's Super-edge magic and antimagic total labeling are presented and verified with examples. JAVA coding for the same algorithms may be written and their performance and complexity may be studied in future to enhance these algorithms.

Conflicts of Interest: Authors have no conflicts of interest to declare.

Acknowledgements: Authors acknowledge their family members and friends for their continuous support.

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